Tax Administration Research Centre

Quasi-Hyperbolic Discounting and Externalities: Can Government Intervention Improve Welfare?

TARC DISCUSSION PAPER 028 - 19

CHRISTOS KOTSOGIANNIS, TARC, UNIVERSITY OF EXETER
ROBERT SCHWAGER, GEORG-AUGUST-UNIVERSITÄT, GÖTTINGEN

University of Exeter | Business School

Economic & Social Research Council

w www.tarc.exeter.ac.uk
e tarc@exeter.ac.uk
t +44 (0) 1392 723464

TARC is an ESRC Research Centre

QUASI-HYPERBOLIC DISCOUNTING AND EXTERNALITIES: CAN GOVERNMENT INTERVENTION IMPROVE WELFARE?

by

Christos Kotsogiannis[a] and Robert Schwager[b]

This version: October 1, 2019

Abstract: The recent literature has emphasized that government intervention when consumers have quasi-hyperbolic preferences ('bias for the present') over consumption is *not welfare-enhancing*. This paper introduces a market imperfection (which takes the form of a negative externality) and shows that government intervention *is welfare enhancing* if the market imperfection is sufficiently strong or the consumers' bias for the present is weak. This conclusion holds, interestingly, even if the government and the consumers share the same biased intertemporal preferences.

Keywords: Quasi-hyperbolic preferences; optimal savings; bias for the present; time consistent policy; externalities.

JEL classification: H23, D15, D9.

Acknowledgements: The support of the Economic and Social Research Council (ESRC) is gratefully acknowledged under Grant Ref: ES/S00713X/1. The usual caveat applies.

[a]Tax Administration Research Center (TARC), Department of Economics, University of Exeter Business School, Streatham Court, Rennes Drive, EX4 4PU, England, UK. E-mail: c.kotsogiannis@exeter.ac.uk; and CESIfo, Munich, Germany.

[b]Chair of Public Economics, Georg-August University Göttingen, Platz der Göttinger Sieben 3, D-37073 Göttingen, Germany; phone: +49 551 39-7293, fax: +49 551 39-7417, email: rschwag@uni-goettingen.de

1 Introduction

Experimental and field research in behavioral economics has shown that consumers have a 'bias for the present' in the sense that a consumer's current self action does not agree with the action taken by the future self and her preferences are time inconsistent. Unsurprisingly, understanding the implications of this 'lack of self-control' in choices for public policy has been the focus of the academic literature. One of the old results in this literature[1] is that as consumers choose a consumption plan for present and future periods, given their intertemporal budget constraint, their marginal rate of substitution between consumptions in two given future periods depends on the period at which it is computed. Since there is a bias for the present they will be saving less for future consumption, a decision they would like to change when that future comes. On an aggregate level, this lack of savings translates into insufficient investment and accumulation, measured against a benchmark where agents are not biased for the present, which may call for government intervention to correct the inefficiency.

Krusell et al. (2002), in an important contribution, analyze an economy within which consumers discount the future in a 'quasi-hyperbolic' way when choosing how much to save.[2] Since they cannot commit to future actions (though they would like to) they can be viewed as they are playing a game with their future selves, with whom they disagree about how much to save. Consumers are, however, rational about their 'internal friction' anticipating the savings decisions of future selves, and taking into account how their own current saving, by raising future income, affects future saving. The social planner and consumers share the same time inconsistent preferences with quasi-hyperbolic discounting. Within this set up Krusell et al. (2002) show that not only a benevolent social planner *does not* deliver the same consumption allocation as does a laissez-faire world but it delivers strictly *lower welfare.* The intuition behind this relies on the idea underlying that the social planner sees through the impact of savings on the rate of return thereby choosing to save less than private individuals would under laissez-faire. As more savings and so more capital accumulation is preferable from an unbiased perspective, the decentralized allocation brings the economy closer to the full commitment outcome. The implication of this is that intervention from the social planner (who makes the same biased intertemporal choices) will imply more immediate consumption and therefore less welfare relative to the competitive case. This is a powerful result pointing towards the undesirability of government intervention showing that welfare improvements are not warranted.

[1]See Strotz (1955), Phelps and Pollak (1968) for early contributions and Frederick et al. (2002) for an insightful survey of the issues.

[2]As will be seen shortly below this is equivalent to shifting uniformly all future utilities (that is, from the date the decision is taken) by a factor β.

The preceding discussion naturally raises the issue of whether the conclusion drawn regarding the undesirability of government intervention still holds in the presence of market imperfections. There are of course many market imperfections one could conceivably think of but to keep things tractable[3] the focus will be on externalities arising as a byproduct of output affecting consumer utility. The aim of the paper is to explore two ultimately interrelated questions: a) How does the bias for the present interact with a negative output-induced externality?, and b) are there circumstances in which government intervention is welfare enhancing? This is, clearly, an important (and general) perspective capturing concerns that relate directly to the role of government in correcting inefficiencies. To address these issues use is made of the model of Krusell et al. (2002), appropriately modified for the issues at hand.

More specifically, the framework introduces a negative externality which arises as a by-product of economic output and characterizes the recursive competitive equilibrium among the sequence of selves of private consumers without any government intervention. Since in this framework, the externality emanates from production and ultimately from investment, the bias for the present, by reducing savings, mitigates the damage brought about by the externality. Thus, in response to question a) above, the model shows that a bias for the present and a production externality cause the outcome to deviate in different directions from the allocation in an ideal world without bias and market imperfection. The competitive equilibrium is contrasted with the equilibrium in a game among a sequence of planner's selves who control saving and investment, distinguishing between a planner who shares the individuals' biased preferences and planner who chooses policies so as to maximize a utility function with geometric discounting. Looking for mnemonic labels we conveniently call the former 'biased' and the latter 'unbiased' planner. It is shown that in the presence of externality the biased planner saves even *less* than she would under no externality.[4] The reason for this is intuitive: The planner internalizes the negative externality and, therefore, has an additional reason to reduce savings further, an incentive which aggravates the effect of controlling the interest rate mentioned above.

The conclusion derived relies on a view for the role of government different from the one which is prevalent in public policy approaches to behavioral failures which views the role of the government as correcting the bias of the individuals.[5] This is a valuable

[3]And to stay within the topical issue of output-induced environmental externalities. Examples of environmental externalities abound: air pollution and congestion to name two. One can think of instances where the externality is on the production possibilities of the economy, affecting, for example, productivity. The precise form of the externality is however not the focus here.

[4]As in Krusell et al. (2002).

[5]Research has examined which government interventions can induce individuals to behave as if they were unbiased. See for example Gruber and Köszegi (2004); O'Donoghue and Rabin (2003, 2006); Thaler and Sunstein (2003); Aronsson and Thunström (2008).

normative benchmark, since, as Gruber and Köszegi (2001, p. 1287) observe, individuals would choose this policy if they could postpone implementation to the next period, and could bind future decision makers. From a positive point of view, however, it is unclear whether a real-world government acts in this way. The political actors will be chosen from the set of citizens, or will be responsible to them, so that, in a democratic society, the government will likely represent individuals' current preferences. Moreover, in realistic political settings the current government cannot easily bind successors, and hence policies must be optimal in every period. The approach therefore taken here contrasts the prevalent view of government with a second one, where the government is assumed to share the biased preferences of individuals, applies the same discounting as the citizens it serves, and cannot bind future governments.

In line with the two views of government outlined in the preceding paragraph, the analysis distinguishes between 'unbiased' welfare, where equilibrium utilities are evaluated with geometric discounting, and 'biased' welfare, where utilities are evaluated in the same way as the current self would evaluate them, including quasi-hyperbolic discounting. According to the first criterion, the bias for the present is considered a failure and therefore should not count when outcomes are judged normatively. According to the second criterion, outcomes should be judged according to the preferences which determine the agents' actions. It should be emphasized that while use is made, for convenience, of the labels unbiased and biased for these criteria, the analysis does not take a normative stand on which one is to be preferred.[6] Instead, it analyzes the welfare consequences for both criteria following government intervention.

The results show that government intervention improves welfare according to both criteria if and only if the externality is sufficiently strong or the bias for the present is weak. When the externality is very important, the competitive equilibrium (laissez-faire) leads to over-saving relative to the welfare maximizing choice, and hence the downward bias on savings exerted by the planner is beneficial. Conversely, when bias for the present' is very pronounced, the planner's savings rate is far too low, whereas the market will save a little less, or only somewhat more, than what would be efficient. Moreover, we find that the range of parameters where government intervention is beneficial is larger when one uses the biased than when one uses the unbiased criterion. This is a consequence of the larger weight the unbiased criterion gives to future payoffs, which argues for larger savings. Hence, the unbiased criterion evaluates the low savings outcome produced by the biased planner even worse than the biased criterion. Altogether, in response to question b)

[6] In the behavioral economics literature, the first view appears to be more popular. See, for example, Thaler and Sunstein (2003, p. 173) who emphasize that revealed preferences do not always equate with welfare. See also O'Donoghue and Rabin (2006, footnote 12, p. 1829). In contrast, Krusell et al. (2002) and Karp (2005) focus on the second welfare criterion.

above, these results show, on the one hand, that one cannot expect a government which, realistically, shares the biased objective of current citizens and cannot commit future policy makers, will implement measures which correct for the bias. On the other hand, we see that, in contrast to the finding by Krusell et al. (2002), even when government is biased in this sense, there is room for welfare improving intervention if a market failure, like a negative externality, is present.

Brief literature review There is a significant literature that addresses the implication of the bias for present, dating back to the contributions of Strotz (1955) and Phelps and Pollak (1968), who first formalized quasi-hyperbolic discounting and its implications for economic outcomes. A general mathematical analysis of the infinite-horizon decision problem of a quasi-hyperbolically discounting consumer is presented by Harris and Laibson (2001). Further theoretical contributions include, among others, Azfar (1999), who relates declining discount rates to uncertainty, Herings and Rohde (2006), who extend the concept of general competitive equilibrium to economies where agents have time-inconsistent preferences, and Salanié and Treich (2006), who emphasize the distinction between additional discounting and self-control problems. Particular attention has been paid to savings decisions by present-biased consumers, which are also in the center of the present paper. Laibson (1998) shows that several features of observed savings behavior, such as the absence of precautionary saving, asset specific marginal propensities to consume, and accumulation in illiquid assets can be explained by the quasi-hyperbolic discounting model. Similarly, Angeletos et al. (2001) simulate behavior under geometric and quasi-hyperbolical discounting and conclude that the latter matches data better than the former. Diamond and Köszegi (2003) consider the incentives for earlier selves to affect retirement decisions through changes in savings, Benartzi and Thaler (2007) discuss heuristics in savings decisions for retirement, and Gustman and Steinmeier (2012) analyze how structural elements of pension systems affect savings and retirement decisions of individuals with quasi-hyperbolic preferences.

For policy, instruments which aim at overcoming under-saving compared to the unbiased benchmark are of significant interest. In this line of research, Laibson (1997) analyzes the purchase of an illiquid asset as a commitment device, and Thaler and Benartzi (2004) report evidence on a program which offers employees the opportunity to commit future pay rises to a retirement savings plan. Malin (2008) shows that a savings floor does not necessarily increase welfare when general equilibrium effects on the interest rate are taken into account, and Andersen and Bhattacharya (2011) show that a pay-as-you-go pension system is only optimal if quasi-hyperbolic discounting is sufficiently strong. Integrating savings decisions by consumers with quasi-hyperbolic preferences into a model with internationally mobile capital, Aronsson and Sjögren (2014) show that optimal policies to correct for the bias for the present differ between large open, small open, and closed economies. Finally, Karp (2005) has analyzed quasi-hyperbolic preferences in relation to environmental issues. In his model, flow emissions which contribute to a stock of pollutant are controlled by a sequence of quasi-hyperbolically discounting regulators. Karp (2005) shows that for additively separable preferences, a planner with commitment power chooses a trajectory of emissions which eventually leads to a lower stock of pollutant. Moreover, in almost all equilibria, reducing the long run stock of pollutant would improve welfare.

The present paper adds to this literature by combining quasi-hyperbolic discounting and an output-induced externality in a unified model. Moreover, unlike most of the literature, we model a government which pursues the preferences of current individuals showing that government intervention can be useful in such a context if market failure is important.

The structure of the paper is as follows. Section 2 sets out the model, and Section 3 analyzes the competitive equilibrium and the planner's choice. In Section 4, we derive welfare maximizing savings rates, compare these to the allocation implemented under laissez-faire and by the planner, and evaluate whether the planner's intervention improves welfare. Section 5 summarizes and concludes. Proofs and longer derivations are relegated to the Appendix.

2 The model

The model is familiar from Krusell et al. (2002) appropriately modified to deal with the issue at hand. There is an infinitely lived consumer who derives utility from consumption, denoted by C, and suffers disutility from a non-tradeable negative externality, denoted by D, at different dates. Time begins at 0, is discrete and infinite and there is no uncertainty.

Utility per period u_t, $t = 0, 1, 2, ...$, is additively separable between consumption and externality and given by

$$u_t = \log(C_t) - \gamma \log(D_t),$$

where γ, with $0 \leq \gamma < 1$, measures the extent of damage created by the externality. Preferences are time-additive and take the form

$$\begin{aligned} U_o &= u_o + \beta\left(\delta u_1 + \delta^2 u_2 + \delta^3 u_3 + ...\right) \\ U_1 &= u_1 + \beta\left(\delta u_2 + \delta^2 u_3 + ...\right) \\ U_2 &= u_2 + \beta\left(\delta u_3 + ...\right) \end{aligned} \tag{1}$$

where δ, $0 < \delta < 1$, is a standard discount factor, and $0 < \beta \leq 1$, as noted earlier, represents additional discounting between the period of decision making and later periods.

If $\beta < 1$ there is a bias towards immediate consumption, and discounting is 'quasi-hyperbolic'.[7] It is clear that with $\beta < 1$ preferences are time inconsistent: at date $t-1$, the trade-off between dates t and $t+1$ is perceived differently than at date t. The point here being that the consumer's self at time t and her self n periods after disagree on the value of consumption in period $t+n$ relative to consumption at date $t+n+1$. For $\beta = 1$ the model reduces to geometric discounting and so the standard inter-temporal choice model where there is no bias for the present. It will be further assumed that there is no technology consumers can use to commit to future consumption levels.

Following Krusell et al. (2002) the consumer rationally understands that her preferences will change as time goes by. The current decision is therefore made taking this into account. The implication of this is that decision-making is modelled as a dynamic game, with the agent's current and future selves as players. The focus is on (first-order) Markov equilibria and so at any moment in time no histories are assumed to matter beyond what is summarised in the current stock of wealth held by the agent.[8]

Production is Cobb-Douglas with full depreciation of capital. The resource constraint simply states that consumption and savings (investment) in a given period must equal output in that period and so

$$C + k' = Ak^{\alpha}\,, \tag{2}$$

where k' denotes investment which, due to full depreciation, equals next period's capital stock, k denotes current capital stock, α, with $0 < \alpha < 1$, is the income share of capital, and $A > 0$ represents the exogenous stock of knowledge.

To capture the externality in a simple way, it is assumed that one unit of output generates on unit of emissions and thus[9]

$$D = Ak^{\alpha}\,.$$

[7] If $\beta > 1$ then there is a bias towards future consumption. This is a case which is not analysed.

[8] The analysis rules out trigger-strategy equilibria of the type studied in Laibson (1994) and Bernheim et al. (1999). Moreover, Markov equilibria in games among a sequence of quasi-hyperbolically discounting selves are typically not unique (see for example Krusell and Smith, Jr., 2003; Karp, 2005; Vieille and Weibull, 2009). The parametric formulation of preferences allows for a closed form solution which eliminates this multiplicity. See Krusell et al. (2002, p. 48).

[9] That is, we model a flow externality. One interpretation is that this is an externality caused by pollution damage (with the stock being completely dissipated at the end of each period). Since the focus is on the inter-temporal trade-off between current and future consumption, which may be biased by quasi-hyperbolic discounting, and how this is affected by negative externalities generated by investment and production, the analysis abstracts from abatement technologies or the choice between industries which differ in the amount of externality created.

Markets are perfectly competitive which implies

$$r = \alpha A k^{\alpha-1}\,, \tag{3}$$
$$w = (1-\alpha) A k^{\alpha}\,, \tag{4}$$

where r is the price of capital and w is the wage rate.

The next section analyzes the competitive equilibrium and the policy chosen by the planner.

3 Recursive equilibrium

3.1 Competitive equilibrium

In the competitive equilibrium the consumer makes his or her decision taking as given the prices as functions of the aggregate capital stock, $\overline{k}$ that is, $r\left(\overline{k}\right)$ and $w\left(\overline{k}\right)$, the law of motion for the aggregate capital stock $\overline{k}' = G\left(\overline{k}\right)$, and the decision rule of her future selves, $g(k, \overline{k})$. She also takes as given the level of the externality. The recursive equilibrium requires then two state variables for the consumer; one for the consumer's own saving decision, k, and one for the economy's savings $\overline{k}$, which determine prices according to (3) and (4).

In any given time period, the current self chooses investment k', taking prices parametrically, so as to solve the problem

$$V_o\left(k,\overline{k}\right) = \max_{k'} \left\{ \log(r\left(\overline{k}\right) k + w\left(\overline{k}\right) - k') - \gamma \log(A\overline{k}^{\alpha}) + \beta\delta V(k', \overline{k}') \right\}\,. \tag{5}$$

This defines the optimal choice rule of the current self given by $k' = \tilde{g}(k, \overline{k})$, which in an equilibrium of the game among the subsequent selves must coincide with expected behavior, that is, $\tilde{g}(k, \overline{k}) = g(k, \overline{k})$ for all $\left(k, \overline{k}\right)$. The continuation value function satisfies

$$V\left(k,\overline{k}\right) = \log\left(r\left(\overline{k}\right) k + w\left(\overline{k}\right) - g(k,\overline{k})\right) - \gamma \log\left(A\overline{k}^{\alpha}\right) + \delta V\left(g(k,\overline{k}), \overline{k}'\right)\,. \tag{6}$$

The laissez-faire solution to the model is given by

Definition 1 *(See Krusell et al., 2002, Definition 1, p. 51.) A recursive competitive equilibrium consists of a decision rule $g\left(k,\overline{k}\right)$, a value function $V\left(k,\overline{k}\right)$, pricing functions $r\left(\overline{k}\right)$ and $w\left(\overline{k}\right)$, and a law of motion for the aggregate capital stock $\overline{k}' = G\left(\overline{k}\right)$ such that:*

1. *Given $V\left(k,\overline{k}\right)$, $g\left(k,\overline{k}\right)$ solves the maximization problem (5);*
2. *Given $g\left(k,\overline{k}\right)$, $V\left(k,\overline{k}\right)$ satisfies (6);*

3. *Firms are price takers and maximize profits, implying that* $r(\overline{k})$ *and* $w(\overline{k})$ *satisfy (3) and (4);*

4. *The law of motion for the aggregate capital stock resulting from the current self's decision is consistent with the law of motion of the aggregate capital stock, that is,* $g(\overline{k},\overline{k}) = G(\overline{k})$.

Equipped with the above definition, the following proposition characterizes the laissez-faire equilibrium in the presence of the externality and quasi-hyperbolic discounting.

Proposition 1 *The recursive competitive equilibrium is given by*

1. $V(k,\overline{k}) = a + b\log\overline{k} + c\log(k+\phi\overline{k})$, *where*

$$c = \frac{1}{(1-\delta)},\ b = \frac{\alpha-1}{(1-\delta\alpha)(1-\delta)} - \frac{\gamma\alpha}{1-\delta\alpha},\ \phi = \frac{(1-\alpha)[1-\delta(1-\beta)]}{\alpha(1-\delta)},$$

2. $g(k,\overline{k}) = \frac{\beta\delta}{1-\delta(1-\beta)} r(\overline{k})\, k$,

3. $G(\overline{k}) = g(\overline{k},\overline{k}) = \frac{\beta\delta\alpha}{1-\delta(1-\beta)} A\overline{k}^{\alpha}$.

Proof. The proof of the proposition is relegated to Appendix A.I. ■

Proposition 1 reconfirms the result by Krusell et al. (2002): This will be the case if $\gamma = 0$. Close inspection of item 1 in Proposition 1 reveals that presence of the externality reduces utility, by the term $(\gamma\alpha)/(1-\delta\alpha) > 0$ in the coefficient b. The savings rate is a constant share of aggregate income $A\overline{k}^{\alpha}$ and given by

$$s_l = \frac{\beta\delta\alpha}{1-\delta(1-\beta)}. \tag{7}$$

Interestingly, it is unaffected by the presence of the externality (and so it is independent of γ).

3.2 The planner's problem

The analysis now turns to the planner's problem. As discussed in the introductory section, the model distinguishes between two 'types' of planner, both of which act in the interest of the representative consumer but differ in what they consider the appropriate objective for the consumer. The first type of planner disregards the bias for the present, in the sense that $\beta = 1$ in (1) whereas the second type considers the preferences of the current self, where β enters (1) with the same value which governs the consumer's choice. For looking for convenient labelling of the two types, and without prejudice regarding the respective normative merits of the two planners' objectives, the first type of planner will be called unbiased and the second one biased.

The analysis now proceeds with the characterization of the biased planner's choices turning to the unbiased shortly after.

Biased planner: The biased planner cannot commit, similar to the consumer, to future actions and anticipates the choices by future planners. Unlike the consumer, however, the planner does not take parametrically prices in choosing aggregate capital k and also takes the externality into account. So in any given time period, the current self (of the planner) chooses investment k' so as to solve the following problem

$$V_{op}(k) = \max_{k'} \ \{\log(Ak^{\alpha} - k') + \beta\delta V(k')\}, \tag{8}$$

which defines the planner's optimal choice rule $k' = \tilde{h}(k)$, and the value function satisfies

$$V_p(k) = \log(Ak^{\alpha} - h(k)) - \gamma\log(Ak^{\alpha}) + \delta V(h(k)). \tag{9}$$

Here, $h(k)$ denotes the anticipated investment rule of future planners, which in an equilibrium must coincide with the current planner's choice, $\tilde{h}(k) = h(k)$.

It can be straightforwardly shown that:

Proposition 2 *The solution to the planner's problem is given by*

1. $V_p(k) = a + b\log k$, *where* $b = \frac{(1-\gamma)\alpha}{1-\delta\alpha}$;
2. $h(k) = \frac{\beta\delta\alpha(1-\gamma)}{1-\delta\alpha[1-\beta(1-\gamma)]}Ak^{\alpha}$.

Proof. The proof parallels the proof of Proposition 1 and is therefore omitted. ∎

Proposition 2 shows that the biased planner also invests a constant share of income given by

$$s_{bp} = \frac{\beta\delta\alpha(1-\gamma)}{1-\delta\alpha[1-\beta(1-\gamma)]}, \tag{10}$$

which, perhaps not surprisingly since this planner internalizes the impact of her decision on the level of the externality, depends on γ (and so the extent of the externality). As expected the planner, even in the absence of an externality and though she has the same preferences as the consumer, chooses a different savings rate than the consumer since she takes into account the impact on the price of capital.

In Proposition 2, the planner directly controls investment. The question then that arises is whether the government can induce private agents to save and invest according to (10) by the appropriate choice of tax instruments. The answer to this is in the affirmative.

Keeping the structure the same, but allowing the planner to have access to proportionate taxes on income τ_y and investment τ_i, it can be shown[10] that:

Proposition 3 *The optimal time consistent tax rates are given by*

$$\tilde{\tau}_i = \frac{\delta(1-\alpha)(1-\beta)+\gamma[1-\delta(1-\beta(1-\alpha))]}{(1-\gamma)[1-\delta(1-\beta(1-\alpha))]}, \tag{11}$$

$$\tilde{\tau}_y = -\frac{\alpha\beta\delta^2(1-\alpha)(1-\beta)+\gamma\alpha\beta\delta[1-\delta(1-\beta(1-\alpha))]}{[1-\delta(1-\beta(1-\alpha))][1-\delta\alpha(1-\beta(1-\gamma))]}. \tag{12}$$

These tax rates implement the savings rate s_{bp} from (10).

Proof. See Appendix A.II. ■

Proposition 3 emphasizes that two instruments are needed for the planner to achieve the level of savings in (10): A combination of an investment tax and a (negative) income tax. What the investment tax does is to correct for the inefficiency in the level of savings (through the return to savings), whereas the (negative) income tax makes sure that the budget is satisfied.

When $\gamma = 0$, and there is no externality, Proposition 4 of Krusell et al. (2002) emerges. Denoting the optimal investment tax rate in the absence of externalities $\tilde{\tau}_i^o$, it is the case that

$$\tilde{\tau}_i - \tilde{\tau}_i^o = \frac{\gamma(1-\delta\alpha)}{(1-\gamma)[1-\delta(1-\beta(1-\alpha))]} > 0,$$

and thus in the presence of an externality the government chooses a higher investment tax rate. Since (11) is increasing in γ, one deduces that, for any admissible α and δ:

Corollary 1 *For any $0 < \beta \leq 1$, the investment tax rate $\tilde{\tau}_i$ increases in the extent of the externality γ.*

The government taxes investment for two reasons: because of the reduction in the marginal product of capital (as in Krusell et al., 2002) and the additional pollution caused tomorrow by an extra dollar of savings today. These two effects point towards the same direction, so that the resulting tax rate is higher than in the special cases $\beta = 1$ or $\gamma = 0$ where only one of the effects is present. In the benchmark case of geometric discounting, when $\beta = 1$, the first effect disappears, and consequently, $\tilde{\tau}_i^o = 0$: Since there is no time inconsistency there is no reason to distort savings decisions. As can be seen from (11), when $\beta = 1$, the resulting tax rate is

$$\tilde{\tau}_i = \gamma/(1-\gamma) > 0.$$

[10]The details of this, being tedious, are relegated to the Appendix.

This is the Pigouvian tax rate which internalizes the pollution externality in a standard model without hyperbolic discounting.

Unbiased planner: The unbiased planner pursues the long run preferences of agents. This comes down to analyzing the planner's optimization problem (8) and (9) with β set equal to one. In this case, following from (10),

$$s_{up} = \frac{\delta\alpha(1-\gamma)}{1-\delta\alpha\gamma} . \tag{13}$$

We turn now to the welfare analysis of competitive equilibrium and planners' choices.

4 Welfare

4.1 Biased and unbiased welfare

We distinguish two welfare criteria. First, we consider welfare according to the preferences which determine agents' actions. That is, we use the preferences of the current self, which incorporate quasi-hyperbolic discounting, as welfare criterion. Second, we consider welfare according to the long-run preferences of individuals, that is, future utilities are discounted geometrically. Paralleling the labeling of the two types of planners, and again without taking a normative position, we label the former criterion as biased welfare, and the latter as unbiased welfare.

The core decision agents or planners have to take in our model is how much to invest. We therefore define both welfare criteria as functions of the current capital stock k and the savings rate s, which is applied in each period. The capital stock evolves according to $k' = sAk^{\alpha}$. Making use of this and (2), the continuation value function (see (6) and (9)) becomes

$$V^*(k;s) = \log\left((1-s)Ak^{\alpha}\right) - \gamma\log\left(Ak^{\alpha}\right) + \delta V\left(sAk^{\alpha};s\right) , \tag{14}$$

which upon iterating forward[11] gives the unbiased welfare as a function of the savings rate

$$\begin{aligned} V^*(k;s) = {} & \frac{1}{1-\delta}\log(1-s) + \frac{\delta\alpha(1-\gamma)}{(1-\delta\alpha)(1-\delta)}\log s \\ & + \frac{\alpha(1-\gamma)}{1-\delta\alpha}\log k + \frac{1-\gamma}{(1-\delta\alpha)(1-\delta)}\log A . \end{aligned} \tag{15}$$

[11]This derivation is contained in Appendix A.III.

The agent's objective in the current period is (see (5) and (8))

$$V_o^*(k;s) = \log\left((1-s)Ak^\alpha\right) - \gamma \log\left(Ak^\alpha\right) + \beta\delta V\left(sAk^\alpha; s\right)\ , \tag{16}$$

and so using (15) in (16) the biased welfare is given by

$$\begin{aligned} V_o^*(k;s) = {} & \frac{1-\delta(1-\beta)}{1-\delta}\log(1-s) + \frac{\beta\delta\alpha(1-\gamma)}{(1-\delta\alpha)(1-\delta)}\log s \\ & + \frac{\alpha(1-\gamma)[1-\delta\alpha(1-\beta)]}{1-\delta\alpha}\log k + (1-\gamma)\left(1+\frac{\beta\delta[1+\alpha(1-\delta)]}{(1-\delta\alpha)(1-\delta)}\right)\log A\,. \end{aligned}$$

The optimal savings rate according to the biased welfare criterion solves the problem $\max_s V_o^*(k;s)$, with first order condition being

$$\frac{\partial V_o^*(k;s)}{\partial s} = -\frac{1-\delta(1-\beta)}{(1-\delta)(1-s)} + \frac{\beta\delta\alpha(1-\gamma)}{(1-\delta\alpha)(1-\delta)s} = 0\,.$$

It is straightforward to verify that $\partial^2 V_o^*/\partial s^2 < 0$ and so $V_o^*(k;s)$ is strictly concave in s and the first order condition is sufficient for a global maximum with the welfare-maximizing savings rate, denoted by s_b^*, given by

$$s_b^* = \frac{\beta\delta\alpha(1-\gamma)}{(1-\delta\alpha)[1-\delta(1-\beta)]+\beta\delta\alpha(1-\gamma)}\,. \tag{17}$$

In the same way, solving $\max_s V^*(k;s)$ yields the savings rate

$$s_u^* = \frac{\delta\alpha(1-\gamma)}{1-\delta\alpha\gamma} \tag{18}$$

which maximizes unbiased welfare.

4.2 Comparing savings rates

The next result shows how the savings rates chosen by the planners, s_{bp} and s_{up}, and achieved under laissez-faire, s_l, compare to the welfare maximizing savings rates s_b^* and s_u^*.

Proposition 4 *For all discount factors and capital shares $0 < \delta, \alpha < 1$, externality parameters $0 \le \gamma < 1$, and short-run discount factors $0 < \beta \le 1$:*

(i) The savings rate chosen by the biased planner is at most as large as the savings rate which maximizes the biased consumer's welfare, which in turn is at most as large as the savings rate which maximizes unbiased welfare, which equals the savings rate

chosen by the unbiased planner: $s_{bp} \leq s_b^* \leq s_u^* = s_{up}$. *The inequalities are strict if and only if* $\beta < 1$.

(ii) *(a)* *The savings rate chosen by the biased planner is at most as large as the laissez-faire savings rate:* $s_{bp} \leq s_l$. *The inequality is strict if and only if* $\beta < 1$ *or* $\gamma > 0$.

(b) *If* $\gamma - \delta\alpha \geq 0$, *then the laissez-faire savings rate exceeds the savings rate which maximizes the biased consumer's welfare:* $s_l > s_b^*$; *if* $\gamma - \delta\alpha < 0$, *then the laissez-faire savings rate is lower than (is equal to, exceeds) the savings rate which maximizes the biased consumer's welfare,* $s_l \lesseqgtr s_b^*$, *if and only if*

$$\beta \lesseqgtr \tilde{\beta}_b(\gamma) := \frac{(\delta\alpha - \gamma)(1 - \delta)}{\delta[\alpha(1 - \delta) + \gamma(1 - \alpha)]} . \tag{19}$$

(c) *The laissez-faire savings rate is lower than (is equal to, exceeds) the savings rate which maximizes unbiased welfare,* $s_l \lesseqgtr s_u^*$, *if and only if*

$$\beta \lesseqgtr \tilde{\beta}_u(\gamma) := \frac{(1 - \gamma)(1 - \delta)}{1 - \delta[1 - \gamma(1 - \alpha)]} . \tag{20}$$

Proof. See Appendix A.IV.

To understand Proposition 4, it is useful to consider first two special cases. When individuals discount geometrically, $\beta = 1$, the model reduces to a standard intertemporal equilibrium model where economic activity causes a negative externality. In this case, according to claim (i), the planners' choices and the two welfare maximizing savings rates coincide, $s_{up} = s_{bp} = s_u^* = s_b^*$.

As can be seen from claim (ii) (a), even when there is no bias towards current consumption the planner's savings rate is lower than in the laissez-faire equilibrium if there is an externality. By saving less, the planner reduces next period's capital stock and, hence, output, which in turn reduces next period's externality. Since, for $\gamma > 0$, we have $\tilde{\beta}_b(\gamma) < 1$ and $\tilde{\beta}_u(\gamma) < 1$, claims (ii) (b) and (c) confirm that this reduction corresponds to a correction of over-saving relative to the welfare maximizing choice, whether one uses the unbiased or biased welfare measure.

In the case $\gamma = 0$ without externality, the model reduces to the one analyzed by Krusell et al. (2002). When there is quasi-hyperbolic discounting, $\beta < 1$, from claim (i), maximizing the current self's welfare requires a lower savings rate than maximizing unbiased welfare, since the biased criterion gives less weight to the future. The biased planner's choice is still lower, since she cannot commit future planners and, hence, can only implement a savings rate which reflects quasi-hyperbolic discounting in every period.

Moreover, we see from claim (ii) (a) that the planner's savings rate is lower than the savings rate in competitive equilibrium, $s_{bp} < s_l$, even without externality. As explained in the introduction, this result stems from the fact that the planner considers the impact of additional savings on the marginal product of capital and, therefore, has less incentive to save. In claims (ii) (b) and (c), we have $\tilde{\beta}_b(0) = \tilde{\beta}_u(0) = 1$. This shows that if there is no externality, the competitive equilibrium necessarily leads to under-saving compared to the welfare maximum. Under-saving is even more severe when the laissez-faire outcome is measured against the unbiased welfare function, which gives future utilities more weight.

We briefly discuss the two opposite extreme cases. If $\gamma \to 1$, then $s_b^* \to 0$, $s_u^* \to 0$, and $s_{bp} \to 0$ but $s_l > 0$. The externality is so dominant that there should not be any savings, and hence hyperbolic discounting does not matter for welfare or the planner. Consequently, savings under laissez-faire, where the externality is ignored, is necessarily excessive.

If $\beta \to 0$, then $s_b^* \to 0$, $s_{bp} \to 0$, and $s_l \to 0$, but $s_u^* > 0$. Here, for the individual and the biased planner, the future has no weight and hence they do not save, in accordance with the biased welfare criterion. In contrast, in the unbiased welfare criterion the future is only discounted geometrically and hence savings should be positive. Thus, in this case, the externality does not matter, and planner and market under-save because of quasi-hyperbolic discounting.

The general case, $\beta < 1$ and $\gamma > 0$, shows how present-bias and externality together shape savings realized under the various institutions considered. Claim (i) is not affected if an externality is introduced into the model where quasi-hyperbolic discounting is present. Planners and welfare criteria internalize the externality essentially in the same way, implying that the order of savings rates does not change. The biased planner saves less than what would be required by both welfare criteria and, hence, induces lower externalities than the welfare maximizing savings rates.[12]

Turning now to the comparison between the competitive equilibrium and the choice of the biased planner in claim (ii) (a), one notices—following from (10)—that $\partial s_{bp}/\partial\gamma = -\delta\beta\alpha\left(1-\delta\alpha\right)/\left[1-\delta\alpha\left(1-\beta\left(1-\gamma\right)\right)\right]^2 < 0$ and so s_{bp} is decreasing in γ. Therefore, the presence of the externality reduces the planner's savings rate further. The planner's present-bias, combined with his or her power to control the interest rate, and his or her desire to internalize the negative externality, point in the same direction, that is, towards

[12] At first sight, this result contrasts with (Karp, 2005, Proposition 2, p. 272) who shows that under additive separable preferences, the game of quasi-hyperbolically discounting regulators ultimately leads to a higher stock of pollutant than the commitment outcome. To understand this difference, observe that in Karp (2005), the planner trades off current benefits of emissions to future damages of the accumulated stock of pollutant. In our model, the trade-off is between current consumption and future income, which entails an externality as a by-product. Commitment in both cases raises the items which generate future utility, which is clean environment in Karp (2005), and income in our model.

a lower savings rate than realized under laissez-faire.

Claim (ii) (b) compares the laissez-faire outcome with the savings rate which maximizes the biased individual's welfare. It shows that a marginal reduction in the savings rate is welfare improving if and only if the externality is sufficiently strong, that is, if $\gamma > \alpha\delta$, or the present-bias is moderate, that is, in (19), $\beta > \tilde{\beta}_b(\gamma)$ holds. Conversely, if the externality is weak or if quasi-hyperbolic discounting is strong, the competitive equilibrium leads to under-saving compared to the welfare maximum. For moderate externalities and short-run discount factors, the laissez-faire savings rate is close to or, by coincidence, even equal to the welfare maximizing one. This arises because the reduction in savings required by time consistency is actually welcome for containing the externality.

According to claim (ii) (c), the laissez-faire savings rate and the savings rate which maximizes unbiased welfare compare in a similar way. Also here, the savings rate realized under competition exceeds (falls short of) the welfare maximizing one if the externality is strong (weak) and/or present-bias is weak (strong). Relating the critical values for the two welfare criteria, one finds that, for any $\gamma > 0$, it holds $\tilde{\beta}_b(\gamma) < \tilde{\beta}_u(\gamma)$. Thus, as illustrated in Figure 1,[13] the range of parameters which leads to under-saving is larger when welfare is evaluated with geometric discounting than when welfare is evaluated according to the current self's preferences. This reflects the higher weight given to future utilities in the unbiased welfare criterion, which requires higher savings.

4.3 Government intervention and welfare

To judge whether the biased government's optimal policy improves welfare, one has to compare welfare levels. We note first that, if the laissez-faire savings rate is smaller than the welfare maximizing savings rate, the impact of quasi-hyperbolic discounting and the lack of commitment depress savings already by more than what is required to internalize the externality. In this case government intervention, which, according to Proposition 4 (ii)(a), reduces the savings rate further, must necessarily be harmful. Thus, the planner's choice can only increase welfare if the competitive market leads to over-saving relative to the respective welfare criterion, that is, if $\beta > \tilde{\beta}_b(\gamma)$ in Proposition 4(ii)(b) or, respectively, $\beta > \tilde{\beta}_u(\gamma)$ in Proposition 4(ii)(c).

This is not sufficient, however. Only if over-saving is strong enough will it outweigh the under-saving implied by the biased planner's choice. To see when this is the case, we next compare the level of welfare reached by laissez-faire and the biased government according to both criteria. Naturally, this is daunting analytically given the logs in the utility functions. Therefore we have taken recourse to numerical simulations.

[13] Figure 1, and Figure 2 of Subsection 4.3, have been drawn with the same parameter values as the ones used in Krusell et al. (2002), $\alpha = 0.36$ and $\delta = 0.95$.

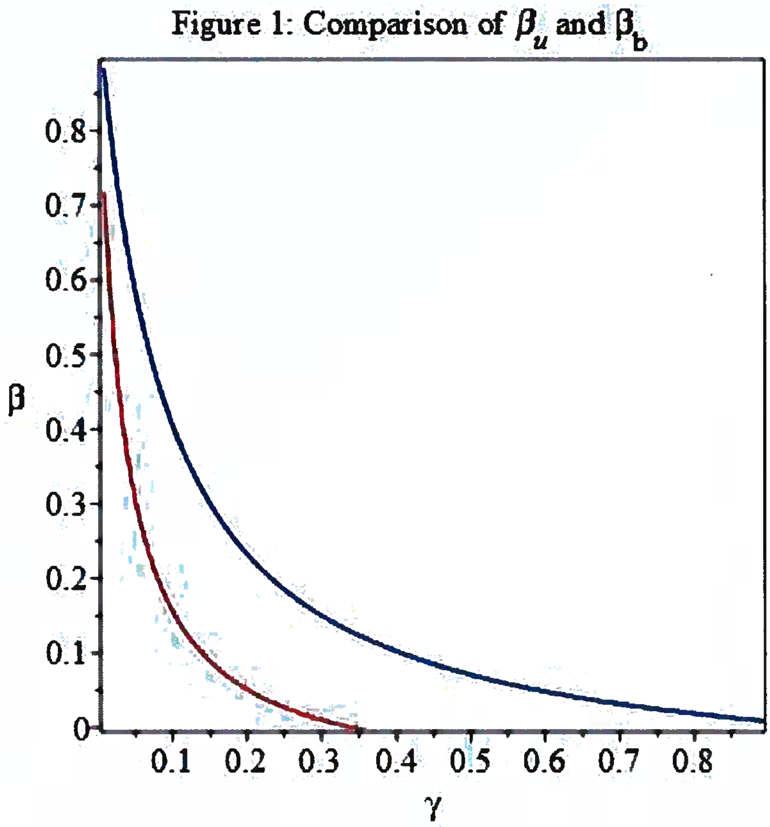

Figure 1: Under- and over-saving in competitive equilibrium relative to biased and unbiased welfare, with $\alpha = 0.36$ and $\delta = 0.95$. For (β, γ) above (below) the red line, which depicts $\tilde{\beta}_b(\gamma)$, the laissez-faire savings rate exceeds (falls short of) the savings rate which maximizes welfare of the current self V_o^*. For (β, γ) above (below) the blue line, which depicts $\tilde{\beta}_u(\gamma)$, the laissez-faire savings rate exceeds (falls short of) the savings rate which maximizes unbiased welfare V^*.

A typical result is displayed in Figure 2. In the figure, we see that for all $0 < \gamma < 1$, there is a critical $\beta^*(\gamma)$ such that unbiased welfare is increased (decreased) by government intervention, that is, $V^*(k; s_{bp}) > (<) V^*(k; s_l)$, if $\beta > (<) \beta^*(\gamma)$. Considering welfare as experienced by the current self, we find that if γ is above a certain threshold, government intervention is preferable to the laissez-faire allocation, no matter what β is. For γ below this threshold, there is a function $\beta_o^*(\gamma)$ such that government intervention increases (decreases) welfare, that is, $V_o^*(k; s_{bp}) > (<) V_o^*(k; s_l)$, if $\beta > (<) \beta_o^*(\gamma)$.

From the figure, one sees that both cutoff lines $\beta^*(\gamma)$ and $\beta_o^*(\gamma)$ are decreasing. This illustrates that the planner's intervention becomes more beneficial when the externality becomes stronger, or when hyperbolic discounting is less pronounced. Moreover, the figure shows that $\beta^*(\gamma) > \beta_o^*(\gamma)$. Thus, there is an intermediate range of parameters where government intervention is beneficial if one applies the current self's welfare criterion, but detrimental if one values welfare according to unbiased preferences. The difference emerges since unbiased welfare gives more weight to the future than the current self's

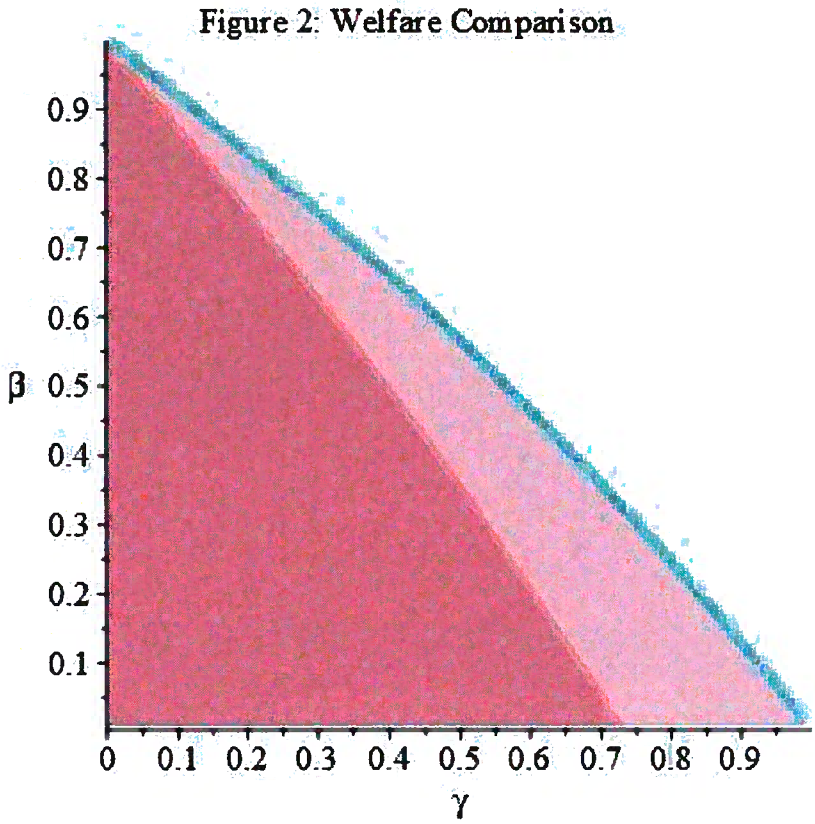

Figure 2: Government intervention and welfare, with $\alpha = 0.36$ and $\delta = 0.95$. In the darkly shaded area, delimited above by the line $\beta_o^*(\gamma)$, biased and unbiased welfare are reduced by moving from laissez-faire to the biased planner's decision. In the unshaded area, delimited below by the line $\beta^*(\gamma)$, welfare improves by the planner's intervention according to both criteria. In the lightly shaded area, biased welfare increases, but unbiased welfare decreases when the competitive outcome is replaced by the biased planner's choice.

preferences, and hence the under-saving induced by the planner is more severe when this criterion is used.

This result shows, on the one hand, that a government which represents the same biased preferences as citizens cannot be expected to correct the present-bias and implement maximal unbiased welfare. This finding contrasts with the view taken in major contributions to behavioral economics such as O'Donoghue and Rabin (2003, 2006), Thaler and Sunstein (2003), or Gruber and Köszegi (2004) who argue in favor of government intervention to correct such biases. Quite the contrary, since the biased government has more powers than individuals in a competitive market, it may, and in our model does, use such powers to even reinforce the impact of behavioral biases. Hence, if the present-bias is sufficiently strong, the biased government may induce a welfare loss compared to the competitive market if welfare is measured by the current self's preferences, and even more so when one considers unbiased welfare.

On the other hand, we see that even the biased government's action can be beneficial when there is another cause of market failure such as a negative externality. Also the biased government aims at correcting this failure and therefore improves welfare if the externality is strong enough, or the bias is weak. This is easier achieved if the welfare criterion allows for quasi-hyperbolic discounting, but even if welfare is measured by unbiased preferences, government intervention will increase welfare in a non-negligible subset of parameters. Thus, the negative result of Krusell et al. (2002), where the planner's choice is always detrimental to welfare, does not in general extend to a richer model where in addition to quasi-hyperbolic discounting, a further motive for public intervention is present.

5 Summary and concluding remarks

This paper has introduced externalities in a framework where consumers have quasi-hyperbolic preferences and so a bias for present consumption. Within such a framework the paper identified conditions under which government intervention is welfare enhancing. In so doing, it distinguished between a government which is unbiased and a government which has the same quasi-hyperbolic preferences as consumers, and welfare was measured both from the biased viewpoint of the current self, and from an unbiased perspective. The results show that even a biased government will improve welfare according to both criteria if the externality is sufficiently important or the bias for the present is not too severe.

This approach suggests two avenues for future research. Firstly, it appears interesting to study the interaction of quasi-hyperbolic preferences and market failures more generally, in contexts different from output-induced externalities. While in the present model a bias for the present mitigates the externality, it is an open question how these elements affect each other in other settings. For example, adverse selection in an insurance market might be more or less pronounced, depending on whether good or bad risks have the stronger bias for the present. Secondly, a host of public policy issues will likely be decided differently by a government with quasi-hyperbolic preferences than by a benevolent, unbiased planner. For example, a biased government may use its market power to actually reduce the price of a good which is detrimental to health (like cigarettes), rather than raise it, as an unbiased planner would do. We hope to have shown that the conclusions derived are instructive and the issues identified merit further investigation.

THIS APPENDIX IS FOR THE ATTENTION OF THE REFEREES AND FOR ONLINE PUBLICATION ONLY.

Appendix

A.I Proof of Proposition 1

Proof. The proof of this proposition proceeds by guessing the functional form of the value function and the law of motion of the aggregate capital and verifying that the optimal decision rule derived using these guesses satisfies the guessed value function and law of motion. Consider the guess

$$V\left(k,\overline{k}\right) = a + b\log\overline{k} + c\log\left(k + \phi\overline{k}\right) \tag{A.1}$$

for the value function and

$$G(\overline{k}) = sA\overline{k}^{\alpha} \tag{A.2}$$

for the law of motion of capital, where are a, b, c, ϕ, s are constants to be determined.

Maximizing (5), making use of (A.1) for the value function $V\left(k', \overline{k}'\right)$, the first order condition is given by

$$\frac{1}{rk + w - k'} = \frac{\beta\delta c}{k' + \phi\overline{k}'},$$

which—after replacing $\overline{k}'$ by the guessed law of motion $G(\overline{k})$, given by (A.2)—can be solved for the optimal investment rule

$$k' = g(k,\overline{k}) = \frac{1}{1+\beta\delta c}\left[\beta\delta c(rk+w) - \phi sA\overline{k}^{\alpha}\right]. \tag{A.3}$$

Substituting (A.3) into (6) and making use of (A.1) and (A.2), one obtains

$$\begin{aligned} V\left(k,\overline{k}\right) = \log&\left(\frac{1}{1+\beta\delta c}\left[(rk+w) + \phi sA\overline{k}^{\alpha}\right]\right) - \gamma\log\left(A\overline{k}^{\alpha}\right) \\ &+ \delta\left[a + b\log\left(sA\overline{k}^{\alpha}\right) + c\log\left(\frac{\beta\delta c}{1+\beta\delta c}\left[(rk+w)+\phi sA\overline{k}^{\alpha}\right]\right)\right]. \end{aligned}$$

Upon making use of (3) and (4), this gives after simplifying

$$\begin{aligned} V\left(k,\overline{k}\right) = &-(1+\delta c)\log(1+\beta\delta c) + (1+\delta c)\log\left(\alpha A\overline{k}^{\alpha-1}k + (1-\alpha)A\overline{k}^{\alpha} + \phi sA\overline{k}^{\alpha}\right) \\ &- \gamma\log A\overline{k}^{\alpha} + \delta a + \delta b\log\left(sA\overline{k}^{\alpha}\right) + \delta c\log(\beta\delta c). \end{aligned}$$

Defining $B \equiv -(1+\delta c)\log(1+\beta\delta c) + \delta a + \delta c\log(\beta\delta c)$, the value function can be re-written as

$$V\left(k,\overline{k}\right) = B + (1+\delta c-\gamma)\log A + \delta b\log(sA) + (1+\delta c)\log\alpha$$
$$+ \left[(1+\delta c)(\alpha-1) - \gamma\alpha + \delta b\alpha\right]\log\overline{k} + (1+\delta c)\log\left(k + \frac{1-\alpha+\phi s}{\alpha}\cdot\overline{k}\right).$$

Now with

$$\begin{aligned} a &= B + (1+\delta c-\gamma)\log A + \delta b\log(sA) + (1+\delta c)\log\alpha\,, \\ b &= (1+\delta c)(\alpha-1) - \gamma\alpha + \delta b\alpha\,, \\ c &= 1+\delta c, \\ \phi &= \frac{1-\alpha+\phi s}{\alpha}\,, \end{aligned}$$

the value function displays the functional form of (A.1). Solving for the constants, it is the case that

$$c = \frac{1}{1-\delta}\,, \tag{A.4}$$

$$b = \frac{\alpha-1}{(1-\delta)(1-\delta\alpha)} - \frac{\gamma\alpha}{1-\delta\alpha}\,, \tag{A.5}$$

$$\phi = \frac{1-\alpha}{\alpha-s}\,. \tag{A.6}$$

In equilibrium, the individual choice of investment must be consistent with the law of motion of aggregate capital stock, that is, $g(\overline{k},\overline{k}) = G(\overline{k})$. Thus, using $k = \overline{k}$ and (A.4)-(A.6) in (A.3) and equating to the guess of the aggregate law of motion (A.2), we have that

$$\frac{1-\delta}{1-\delta(1-\beta)}\left(\frac{\beta\delta}{1-\delta} - \frac{(1-\alpha)s}{\alpha-s}\right) = s\,,$$

with the two solutions $s = 1$ and

$$s = \frac{\beta\delta\alpha}{1-\delta(1-\beta)}\,. \tag{A.7}$$

$s = 1$ cannot be a feasible solution since it would imply that aggregate investment is equal to total output, $G(\overline{k}) = Ak^{\alpha}$, and so consumption would be zero. Inserting (A.7) in (A.6) it is the case that

$$\phi = \frac{(1-\alpha)[1-\delta(1-\beta)]}{\alpha(1-\delta)}\,. \tag{A.8}$$

Making use of (A.7) in the guessed law of motion for capital (A.2) shows that $G(\overline{k})$ is as stated in the proposition. Using c, s, and ϕ, from (A.4), (A.7), and (A.8), in (A.3), one obtains after some manipulations

$$g(k,\overline{k}) = \frac{\beta\delta\alpha}{1-\delta(1-\beta)} A\overline{k}^{\alpha-1} k = \frac{\beta\delta}{1-\delta(1-\beta)} r(\overline{k}) k \,,$$

where the second equality follows upon using (3). ■

A.II Optimal tax policy

Suppose that the government wishes to implement its preferred policy having access to proportionate taxes on income τ_y and investment τ_i. Since it cannot commit to future tax policies, the current government chooses current tax rates $\tilde{\tau}_y$ and $\tilde{\tau}_i$ and anticipates equilibrium tax rates $\tau_y(\overline{k})$ and $\tau_i(\overline{k})$ chosen by future governments, which may depend on the capital stock. Private agents optimize taking these taxes as given, so that the competitive equilibrium depends on tax rates. The following definition characterizes a time consistent tax policy equilibrium.

Definition 2 *A time consistent tax policy equilibrium consists of three elements.*

1. *The behavior along the equilibrium path is as follows:*
   - *Tax outcomes are given by* $\tau(\overline{k}) = (\tau_y(\overline{k}), \tau_i(\overline{k}))$.
   - *Given this tax function, the law of motion for aggregate capital is given by* $G(\overline{k})$.
   - *Given the tax function and the law of motion for aggregate capital, the individual decision rule is given by* $g(k, \overline{k})$.
2. *The laws of motion after one-period deviations of tax rates* $\tilde{\tau} = (\tilde{\tau}_y, \tilde{\tau}_i)$ *for the current period, with future taxes given by the tax outcome functions evaluated at the capital stocks implied by the current tax rates and the implied capital accumulation, are given by the following:*
   - $\tilde{G}(\overline{k}, \tilde{\tau})$ *describes the law of motion for aggregate capital for the one period deviation.*
   - $\tilde{g}(k, \overline{k}, \tilde{\tau})$ *describes the individual decision for the one period deviation.*
3. *We have competitive pricing functions* $r(\overline{k})$ *and* $w(\overline{k})$ *given by (3) and (4).*

These equilibrium elements must satisfy:

1. $\tilde{g}(k,\overline{k},\tilde{\tau})$ *solves the consumer's optimization problem*

$$V_o\left(k,\overline{k},\tilde{\tau}\right) = \max_{k'} \left\{\log\left((rk+w)(1-\tilde{\tau}_y) - k'(1+\tilde{\tau}_i)\right) - \gamma\log\left(A\overline{k}^{\alpha}\right) + \beta\delta V\left(k',\tilde{G}(\overline{k},\tilde{\tau})\right)\right\}, \tag{A.9}$$

and the continuation value function is given by

$$V\left(k,\overline{k}\right) = \log\left((rk+w)(1-\tau_y(\overline{k})) - g(k,\overline{k})(1+\tau_i(\overline{k}))\right) - \gamma\log\left(A\overline{k}^{\alpha}\right) + \delta V\left(g(k,\overline{k}),G(\overline{k})\right). \tag{A.10}$$

2. Consistency between individual and aggregate actions: $\tilde{g}(\overline{k},\overline{k},\tilde{\tau}) = \tilde{G}(\overline{k},\tilde{\tau})$ *which implies that, if* $\tilde{\tau} = \tau(\overline{k})$*, then* $g(\overline{k},\overline{k}) = G(\overline{k})$*.*

3. The government optimizes: $\tau(\overline{k}) = \left(\tau_y(\overline{k}),\tau_i(\overline{k})\right)$ *solves*

$$\begin{aligned} &\max_{\tilde{\tau}_y,\tilde{\tau}_i} \quad V_o(\overline{k},\overline{k},\tilde{\tau}), \\ &\text{subject to} \quad -\tilde{G}(\overline{k},\tilde{\tau})\tilde{\tau}_i = A\overline{k}^{\alpha}\tilde{\tau}_y. \end{aligned} \tag{A.11}$$

Proof of Proposition 3

We start by solving the individual optimization problem (step a) and finding the competitive equilibrium with taxes. As in the proof of Proposition 1, we proceed by guessing the functional form of the value function and the law of motion of the aggregate capital and verifying that the optimal decision rule derived using these guesses satisfies the guessed value function (step b) and law of motion (step c). We then solve for the government's optimal tax rates (step d) and finally show that these tax rates implement the planner's preferred savings rate (step e).

Consider the guess

$$V(k,\overline{k}) = a + b\log\overline{k} + c\log\left(k+\phi\overline{k}\right) \tag{A.12}$$

for the value function and $G(\overline{k}) = sA\overline{k}^{\alpha}$ for the law of motion of capital (for suitable choices of the constants a, b, c, ϕ, s).

(a) Individual optimization. Solving the optimization problem (A.9) and using the guess (A.12) for the value function $V\left(k',\overline{k}'\right)$, we have that

$$\frac{1+\tilde{\tau}_i}{(rk+w)(1-\tilde{\tau}_y) - k'(1+\tilde{\tau}_i)} = \frac{\beta\delta c}{k'+\phi\overline{k}'},$$

which can be solved for the optimal investment rule for the current self

$$k' = \tilde{g}(k, \overline{k}, \tilde{\tau}) = \frac{1}{1+\beta\delta c}\left[\beta\delta c(rk+w)\frac{(1-\tilde{\tau}_y)}{(1+\tilde{\tau}_i)} - \phi\overline{k}'\right]. \tag{A.13}$$

(b) Value function. For the continuation value function, this decision rule is evaluated at the equilibrium tax policy rule, $\tilde{\tau} = \tau(\overline{k})$. Substituting (A.13) into (A.10) and making use of (A.12), one obtains

$$\begin{aligned} V(k,\overline{k}) = &\log\left\{\frac{1}{1+\beta\delta c}\left[(rk+w)(1-\tau_y(\overline{k})) + \phi\overline{k}'(1+\tau_i(\overline{k}))\right]\right\} \\ &- \gamma\log A\overline{k}^{\alpha} + \delta a + \delta b\log\overline{k}' + \delta c\log\left(\frac{\beta\delta c}{1+\beta\delta c}\left[(rk+w)\frac{(1-\tau_y(\overline{k}))}{(1+\tau_i(\overline{k}))} + \phi\overline{k}'\right]\right). \end{aligned} \tag{A.14}$$

In equilibrium, with taxes following $\tau(\overline{k})$, the individual choice of investment must equal next period's aggregate capital. From (A.13), and with $k' = \overline{k}'$, $k = \overline{k}$, $\tilde{\tau} = \tau(\overline{k})$, it is the case that

$$\overline{k}' = \tilde{g}(\overline{k}, \overline{k}, \tau(\overline{k})) = \frac{1}{1+\beta\delta c}\left[\beta\delta c(r\overline{k}+w)\frac{(1-\tau_y(\overline{k}))}{(1+\tau_i(\overline{k}))} - \phi\overline{k}'\right],$$

which upon solving for $\overline{k}'$ yields, making use of (3) and (4), gives

$$\overline{k}' = \frac{\beta\delta c}{(1+\beta\delta c+\phi)}\frac{(1-\tau_y(\overline{k}))}{(1+\tau_i(\overline{k}))}A\overline{k}^{\alpha}. \tag{A.15}$$

Substituting (A.15) into (A.14) and using (3) and (4), we obtain

$$\begin{aligned} V(k,\overline{k}) = &\log\left(\frac{(1-\tau_y(\overline{k}))}{(1+\beta\delta c)}A\overline{k}^{\alpha-1}\left[\alpha k + (1-\alpha)\overline{k} + \phi\frac{\beta\delta c}{(1+\beta\delta c+\phi)}\overline{k}\right]\right) \\ &- \gamma\log\left(A\overline{k}^{\alpha}\right) + \delta a + \delta b\log\left(\frac{\beta\delta c}{(1+\beta\delta c+\phi)}\frac{(1-\tau_y(\overline{k}))}{(1+\tau_i(\overline{k}))}A\overline{k}^{\alpha}\right) \\ &+ \delta c\log\left(\frac{\beta\delta c}{(1+\beta\delta c)}A\overline{k}^{\alpha-1}\frac{(1-\tau_y(\overline{k}))}{(1+\tau_i(\overline{k}))}\left[\alpha k + (1-\alpha)\overline{k} + \phi\frac{\beta\delta c}{(1+\beta\delta c+\phi)}\overline{k}\right]\right) \\ = &\log\left(\frac{(1-\tau_y(\overline{k}))}{(1+\beta\delta c)}A\right) - \gamma\log A + \delta a + \delta b\log\left(\frac{\beta\delta c}{(1+\beta\delta c+\phi)}\frac{(1-\tau_y(\overline{k}))}{(1+\tau_i(\overline{k}))}A\right) \\ &+ \delta c\log\left(\frac{\beta\delta c}{(1+\beta\delta c)}\frac{(1-\tau_y(\overline{k}))}{(1+\tau_i(\overline{k}))}A\right) + \left[(\alpha-1) - \gamma\alpha + \delta b\alpha + \delta c(\alpha-1)\right]\log\overline{k} \\ &+ (1+\delta c)\log\left(\alpha\left[k + \frac{(1-\alpha)(1+\beta\delta c+\phi)+\phi\beta\delta c}{\alpha(1+\beta\delta c+\phi)}\overline{k}\right]\right). \end{aligned} \tag{A.16}$$

Denoting

$$B \equiv \log\left(\frac{(1-\tau_y(\overline{k}))A}{1+\beta\delta c}\right) - \gamma\log A + \delta a + \delta b\left(\log\frac{\beta\delta c}{(1+\beta\delta c+\phi)}\frac{(1-\tau_y(\overline{k}))}{(1+\tau_i(\overline{k}))}A\right)$$
$$+ \delta c\log\left(\frac{\beta\delta c}{(1+\beta\delta c)}\frac{(1-\tau_y(\overline{k}))}{(1+\tau_i(\overline{k}))}A\right),$$

the value function (A.16) can be re-written as

$$V(k,\overline{k}) = B + [(1+\delta c)(\alpha-1) + (\delta b-\gamma)\alpha]\log\overline{k}$$
$$+ (1+\delta c)\log\alpha + (1+\delta c)\log\left[k + \frac{(1-\alpha)(1+\beta\delta c+\phi)+\phi\beta\delta c}{\alpha(1+\beta\delta c+\phi)}\overline{k}\right].$$

With

$$a = B + (1+\delta c)\log\alpha,$$
$$b = (1+\delta c)(\alpha-1) + (\delta b-\gamma)\alpha,$$
$$c = 1+\delta c,$$
$$\phi = \frac{(1-\alpha)(1+\beta\delta c+\phi)+\phi\beta\delta c}{\alpha(1+\beta\delta c+\phi)},$$

and solving[14]

$$b = \frac{\alpha-1}{(1-\delta\alpha)(1-\delta)} - \frac{\gamma\alpha}{1-\delta\alpha}, \tag{A.17}$$
$$c = \frac{1}{1-\delta}, \tag{A.18}$$
$$\phi = \frac{(1-\alpha)(1-\delta(1-\beta))}{\alpha(1-\delta)}, \tag{A.19}$$

one verifies that the value function takes the form guessed in (A.12).

(c) Law of motion. Using (A.17) to (A.19) in (A.15), and observing that in equilibrium, (A.15) must be consistent with the law of motion of capital, so that $\overline{k}' = G(\overline{k}) = sA\overline{k}^{\alpha}$, we obtain with (7)

$$G(\overline{k}) = sA\overline{k}^{\alpha} = \frac{(1-\tau_y(\overline{k}))}{(1+\tau_i(\overline{k}))}s_lA\overline{k}^{\alpha} = g(\overline{k},\overline{k}).$$

[14]In (A.19), we discard the solution $\phi = -1$ since the argument under the last log in (A.12) must be positive.

Solving (A.13) and using (A.17) to (A.19) yields the current self's investment rule

$$k' = \tilde{g}(k, \overline{k}, \tilde{\tau}) = \frac{\beta\delta\alpha}{1-\delta(1-\beta)} \left(\alpha A\overline{k}^{\alpha-1}k + (1-\alpha)A\overline{k}^{\alpha}\right) \frac{(1-\tilde{\tau}_y)}{(1+\tilde{\tau}_i)}$$

for the one-period deviation in taxes $\tilde{\tau}$. Equating this to the aggregate law of motion $\tilde{G}(\overline{k}, \tilde{\tau})$ and using the equilibrium condition $k = \overline{k}$, it follows with (7)

$$\tilde{G}(\overline{k}, \tilde{\tau}) = \frac{\beta\delta\alpha}{1-\delta(1-\beta)} \frac{(1-\tilde{\tau}_y)}{(1+\tilde{\tau}_i)} A\overline{k}^{\alpha} = \frac{(1-\tilde{\tau}_y)}{(1+\tilde{\tau}_i)} s_l A\overline{k}^{\alpha}. \tag{A.20}$$

(d) Choice of tax rates. Inserting (A.20) into (A.9), we find, using the equilibrium condition $\tilde{g}(\overline{k}, \overline{k}, \tilde{\tau}) = \tilde{G}(\overline{k}, \tilde{\tau})$, (A.12), and the pricing functions (3) and (4),

$$\begin{aligned} V_o(\overline{k}, \overline{k}, \tilde{\tau}) = & \log\left((1-\tilde{\tau}_y)(1-s_l)A\overline{k}^{\alpha}\right) - \gamma \log\left(A\overline{k}^{\alpha}\right) + \beta\delta a \\ & + \beta\delta b \log\left(\frac{(1-\tilde{\tau}_y)}{(1+\tilde{\tau}_i)} s_l A\overline{k}^{\alpha}\right) + \beta\delta c \log\left(\frac{(1-\tilde{\tau}_y)}{(1+\tilde{\tau}_i)}(1+\phi)s_l A\overline{k}^{\alpha}\right). \end{aligned}$$

Maximizing this function with respect to the tax rates $\tilde{\tau}_y$ and $\tilde{\tau}_i$ is equivalent to maximizing

$$[1+\beta\delta(b+c)]\log(1-\tilde{\tau}_y) - \beta\delta(b+c)\log(1+\tilde{\tau}_i). \tag{A.21}$$

Using the law of motion of capital (A.20), the government budget constraint (A.11) reads $-(1-\tilde{\tau}_y)s_l\tilde{\tau}_i = \tilde{\tau}_y(1+\tilde{\tau}_i)$. Solving for the income tax rate, we get

$$\tilde{\tau}_y = -\frac{s_l\tilde{\tau}_i}{1+\tilde{\tau}_i(1-s_l)}. \tag{A.22}$$

Substituting $\tilde{\tau}_y$ in (A.21) and simplifying, one obtains

$$\log(1+\tilde{\tau}_i) - [1+\beta\delta(b+c)]\log(1+\tilde{\tau}_i(1-s_l)).$$

Maximizing this with respect to $\tilde{\tau}_i$ gives the optimal investment tax rate

$$\tilde{\tau}_i = -1 + \frac{s_l}{(1-s_l)\beta\delta(b+c)}. \tag{A.23}$$

Putting the optimal investment tax rate back into (A.22), we find the optimal income tax rate

$$\tilde{\tau}_y = \frac{(1-s_l)\beta\delta(b+c) - s_l}{(1-s_l)[1+\beta\delta(b+c)]}. \tag{A.24}$$

From (A.17) and (A.18), we have

$$b + c = \frac{\alpha(1-\gamma)}{1-\delta\alpha}. \tag{A.25}$$

Upon substitution of $b+c$, from (A.25), and s_l, from (7), in (A.23), one finds the optimal investment tax rate as in (11). Using the same substitutions in (A.24) and after collecting terms, the optimal income tax rate is as in (12).

(e) Savings rate. From (A.23) and (A.24) we find

$$1+\tilde{\tau}_i = \frac{s_l}{(1-s_l)\beta\delta(b+c)},$$
$$1-\tilde{\tau}_y = \frac{1}{(1-s_l)[1+\beta\delta(b+c)]}.$$

With these equations, (7), and (A.25), the savings rate s in (A.20) is

$$s = \frac{1-\tilde{\tau}_y}{1+\tilde{\tau}_i}s_l = \frac{\beta\delta(b+c)}{1+\beta\delta(b+c)} = \frac{\beta\delta\alpha(1-\gamma)}{1-\delta\alpha[1-\beta(1-\gamma)]} = s_{bp}$$

as claimed. ■

A.III Derivation of (15)

With savings rate s, the sequence of capital stocks in the first, second, third, ... period after the present one is

$$\begin{aligned} k' &= sAk^{\alpha}, \\ k'' &= sAk'^{\alpha} = (sA)^{1+\alpha}k^{\alpha^2}, \\ k''' &= sAk''^{\alpha} = (sA)^{1+\alpha+\alpha^2}k^{\alpha^3}, \\ &\dots \end{aligned} \tag{A.26}$$

From (14), welfare is the infinite sum

$$\begin{aligned} V^*(k;s) = {} & \log((1-s)Ak^{\alpha}) - \gamma\log(Ak^{\alpha}) + \delta\big[\log((1-s)Ak'^{\alpha}) - \gamma\log(Ak'^{\alpha})\big] \\ & + \delta^2\big[\log((1-s)Ak''^{\alpha}) - \gamma\log(Ak''^{\alpha})\big] + \dots . \end{aligned}$$

Inserting capital stocks from (A.26), one has

$$\begin{aligned} V^*(k;s) = {} & \log((1-s)Ak^{\alpha}) - \gamma\log(Ak^{\alpha}) \\ & + \delta\left[\log\left((1-s)A(sA)^{\alpha}k^{\alpha^2}\right) - \gamma\log\left(A(sA)^{\alpha}k^{\alpha^2}\right)\right] \\ & + \delta^2\left[\log\left((1-s)A(sA)^{\alpha+\alpha^2}k^{\alpha^3}\right) - \gamma\log\left(A(sA)^{\alpha+\alpha^2}k^{\alpha^3}\right)\right] \\ & + \dots . \\ = {} & \left(1+\delta+\delta^2+\dots\right)\log((1-s)A) - \gamma\left(1+\delta+\delta^2+\dots\right)\log A \\ & + (1-\gamma)\left[\delta\alpha+\delta^2\left(\alpha+\alpha^2\right)+\delta^3\left(\alpha+\alpha^2+\alpha^3\right)+\dots\right]\log(sA) \end{aligned}$$

$$
\begin{aligned}
&+\alpha(1-\gamma)\left(1+\delta\alpha+\delta^2\alpha^2+...\right)\log k \\
=&\ \frac{1}{1-\delta}\log(1-s)+\frac{1-\gamma}{1-\delta}\log A \\
&+\delta\alpha(1-\gamma)\left[1+\delta\left(1+\alpha\right)+\delta^2\left(1+\alpha+\alpha^2\right)+...\right]\log(sA)+\frac{\alpha(1-\gamma)}{1-\delta\alpha}\log k \\
=&\ \frac{1}{1-\delta}\log(1-s)+\frac{1-\gamma}{1-\delta}\log A \\
&+\delta\alpha(1-\gamma)\left(1+\delta\alpha+\delta^2\alpha^2+...\right)\left(1+\delta+\delta^2+...\right)\log(sA)+\frac{\alpha(1-\gamma)}{1-\delta\alpha}\log k \\
=&\ \frac{1}{1-\delta}\log(1-s)+\frac{1-\gamma}{1-\delta}\log A \\
&+\delta\alpha(1-\gamma)\cdot\frac{1}{1-\delta\alpha}\cdot\frac{1}{1-\delta}\cdot\log(sA)+\frac{\alpha(1-\gamma)}{1-\delta\alpha}\log k \\
=&\ \frac{1}{1-\delta}\log(1-s)+\frac{\delta\alpha(1-\gamma)}{(1-\delta\alpha)(1-\delta)}\log s+\frac{1-\gamma}{(1-\delta\alpha)(1-\delta)}\log A+\frac{\alpha(1-\gamma)}{1-\delta\alpha}\log k\,,
\end{aligned}
$$

hence (15).

A.IV Proof of Proposition 4

Claim (i). From (10) and (17), $s_{bp} \leq s_b^*$ is equivalent to

$$
\frac{\beta\delta\alpha(1-\gamma)}{1-\delta\alpha[1-\beta(1-\gamma)]} \leq \frac{\beta\delta\alpha(1-\gamma)}{(1-\delta\alpha)[1-\delta(1-\beta)]+\beta\delta\alpha(1-\gamma)}\,.
$$

With $\alpha, \beta, \delta > 0$ and $\gamma < 1$ this reduces to

$$
\begin{aligned}
(1-\delta\alpha)[1-\delta(1-\beta)]+\beta\delta\alpha(1-\gamma) &\leq 1-\delta\alpha[1-\beta(1-\gamma)] \\
1-\delta\alpha-(1-\delta\alpha)\delta(1-\beta)+\beta\delta\alpha(1-\gamma) &\leq 1-\delta\alpha+\beta\delta\alpha(1-\gamma) \\
0 &\leq (1-\delta\alpha)\delta(1-\beta)\,.
\end{aligned}
$$

With $\alpha, \delta < 1$, this is equivalent to $\beta \leq 1$. In the same way, one proves that $s_{bp} < s_b^*$ is equivalent to $\beta < 1$. From (17) and (18), $s_b^* \leq s_u^*$ is equivalent to

$$
\frac{\beta\delta\alpha(1-\gamma)}{(1-\delta\alpha)[1-\delta(1-\beta)]+\beta\delta\alpha(1-\gamma)} \leq \frac{\delta\alpha(1-\gamma)}{1-\delta\alpha\gamma}\,.
$$

With $\alpha, \beta, \delta > 0$ and $\gamma < 1$ this reduces to

$$
\begin{aligned}
\beta(1-\delta\alpha\gamma) &\leq (1-\delta\alpha)[1-\delta(1-\beta)]+\beta\delta\alpha(1-\gamma) \\
\beta[(1-\delta\alpha\gamma)-\delta(1-\delta\alpha)-\delta\alpha(1-\gamma)] &\leq (1-\delta\alpha)(1-\delta) \\
\beta(1-\delta\alpha)(1-\delta) &\leq (1-\delta\alpha)(1-\delta)\,.
\end{aligned}
$$

With $\alpha, \delta < 1$, this reduces to $\beta \le 1$. In the same way, one proves that $s_b^* < s_u^*$ is equivalent to $\beta < 1$. The equation $s_u^* = s_{up}$ is immediate from (13) and (18).

Claim (ii). **(a)** From (7) and (10), $s_{bp} \le s_l$ is equivalent to

$$\frac{\beta\delta\alpha(1-\gamma)}{1-\delta\alpha[1-\beta(1-\gamma)]} \le \frac{\beta\delta\alpha}{1-\delta(1-\beta)}.$$

With $\alpha, \beta, \delta > 0$ and $\delta < 1$, this reduces to

$$\begin{aligned}(1-\gamma)[1-\delta(1-\beta)] &\le 1-\delta\alpha[1-\beta(1-\gamma)]\\ 1-\gamma-\delta(1-\beta)+\gamma\delta(1-\beta) &\le 1-\delta\alpha(1-\beta)-\delta\alpha\beta\gamma\\ 0 &\le \delta(1-\beta)-\delta\alpha(1-\beta)+\gamma[1-\delta(1-\beta)-\delta\alpha\beta]\\ 0 &\le \delta(1-\beta)(1-\alpha)+\gamma[1-\delta(1-\beta(1-\alpha))],\end{aligned}$$

which is true for $\alpha, \beta \le 1$ and $\gamma \ge 0$. Replacing '$\le$' by '$<$' in this argument shows that $s_{bp} < s_l$ is equivalent to

$$\delta(1-\alpha)(1-\beta)+\gamma[1-\delta(1-\beta(1-\alpha))] > 0 \tag{A.27}$$

if $\alpha, \beta, \delta > 0$ and $\delta < 1$ hold. Inequality (A.27) follows from $\alpha < 1$ and $\beta < 1$, or from $\gamma > 0$.

(b) From (7) and (17), $s_l \lesseqgtr s_b^*$ is equivalent to

$$\frac{\beta\delta\alpha}{1-\delta(1-\beta)} \lesseqgtr \frac{\beta\delta\alpha(1-\gamma)}{(1-\delta\alpha)[1-\delta(1-\beta)]+\beta\delta\alpha(1-\gamma)}.$$

With $\alpha, \beta, \delta > 0$ and $\delta < 1$, this reduces to

$$\begin{aligned}(1-\delta\alpha)[1-\delta(1-\beta)]+\beta\delta\alpha(1-\gamma) &\lesseqgtr (1-\gamma)[1-\delta(1-\beta)]\\ (1-\delta\alpha)(1-\delta)+(1-\delta\alpha)\beta\delta+\beta\delta\alpha(1-\gamma) &\lesseqgtr (1-\gamma)(1-\delta)+(1-\gamma)\beta\delta\\ \beta\delta[(1-\delta\alpha)+\alpha(1-\gamma)-(1-\gamma)] &\lesseqgtr (1-\delta)[(1-\gamma)-(1-\delta\alpha)],\end{aligned}$$

which can be re-written as

$$\beta\delta[\alpha(1-\delta)+\gamma(1-\alpha)] \lesseqgtr (\delta\alpha-\gamma)(1-\delta). \tag{A.28}$$

If $\gamma \ge \delta\alpha$, then the right-hand-side of (A.28) is non-positive. Hence, with $\beta > 0$, $s_l > s_{bp}$ follows. If $\gamma < \delta\alpha$, solving (A.28) for β yields (19).

(c) From (7) and (18), $s_l \lesseqgtr s_u^*$ is equivalent to

$$\frac{\beta\delta\alpha}{1-\delta(1-\beta)} \lesseqgtr \frac{\delta\alpha(1-\gamma)}{1-\delta\alpha\gamma}\,.$$

With $\alpha, \delta > 0$ and $\delta < 1$ this reduces to

$$\begin{aligned}
\beta(1-\delta\alpha\gamma) &\lesseqgtr (1-\gamma)[1-\delta(1-\beta)] \\
\beta(1-\delta\alpha\gamma) &\lesseqgtr (1-\gamma)(1-\delta) + \beta\delta(1-\gamma) \\
\beta(1-\delta\alpha\gamma-\delta+\delta\gamma) &\lesseqgtr (1-\gamma)(1-\delta) \\
\beta\{1-\delta[1-\gamma(1-\alpha)]\} &\lesseqgtr (1-\gamma)(1-\delta)\,.
\end{aligned}$$

Solving for β yields (20). ∎

Disclosure statement

Funding: The research by Christos Kotsogiannis was funded by Economic and Social Research Council (ESRC) under Grant Ref: ES/S00713X/1.

Conflicts of interest: The authors declare that they have no conflict of interest.

References

Andersen, T. M., Bhattacharya, J., 2011. On myopia as rationale for social security. Economic Theory 47 (1), 135–158.

Angeletos, G.-M., Laibson, D., Repetto, A., Tobacman, J., Weinberg, S., 2001. The hyperbolic consumption model: Calibration, simulation, and empirical evaluation. Journal of Economic Perspectives 15 (3), 47–68.

Aronsson, T., Sjögren, T., 2014. Tax policy and present-biased preferences: Paternalism under international capital mobility. Journal of Economic Behavior & Organization 106, 298–316.

Aronsson, T., Thunström, L., 2008. A note on optimal paternalism and health capital subsidies. Economics Letters 101 (3), 241 – 242.

Azfar, O., 1999. Rationalizing hyperbolic discounting. Journal of Economic Behavior & Organization 38, 245–252.

Benartzi, S., Thaler, R. H., 2007. Heuristics and biases in retirement savings behavior. Journal of Economic Perspectives 21 (3), 81–104.

Bernheim, B. D., Ray, D., Yeltekin, Ş., May 1999. Self-control, saving, and the low asset trap.
URL `http://www.econ.nyu.edu/user/debraj/Papers/selfcontrol.pdf`

Diamond, P., Köszegi, B., 2003. Quasi-hyperbolic discounting and retirement. Journal of Public Economics 87 (910), 1839 – 1872.

Frederick, S., Loewenstein, G., O'Donoghue, T., 2002. Time discounting and time preference: A critical review. Journal of Economic Literature 40 (2), 351–401.

Gruber, J., Köszegi, B., 2001. Is addiction rational? Theory and evidence. Quarterly Journal of Economics 116, 1261–1303.

Gruber, J., Köszegi, B., 2004. Tax incidence when individuals are time-inconsistent: The case of cigarette excise taxes. Journal of Public Economics 88 (9–10), 1959 – 1987.

Gustman, A. L., Steinmeier, T. L., 2012. Policy effects in hyperbolic vs. exponential models of consumption and retirement. Journal of Public Economics 96, 465473.

Harris, C., Laibson, D., 2001. Dynamic choices of hyperbolic consumers. Econometrica 69 (4), 935–957.

Herings, P. J.-J., Rohde, K., 2006. Time-inconsistent preferences in a general equilibrium model. Economic Theory 29 (3), 591–619.

Karp, L., 2005. Global warming and hyperbolic discounting. Journal of Public Economics 89 (23), 261 – 282.

Krusell, P., Kuruşçu, B., Smith, Jr., A. A., 2002. Equilibrium welfare and government policy with quasi-geometric discounting. Journal of Economic Theory 105, 42–72.

Krusell, P., Smith, Jr., A. A., 2003. Consumption-savings decisions with quasi-geometric discounting. Econometrica 71 (1), 365–375.

Laibson, D., 1994. Essays in hyperbolic discounting. Ph.D. dissertation.

Laibson, D., 1997. Golden eggs and hyperbolic discounting. Quarterly Journal of Economics 112 (2), 443–478.

Laibson, D., 1998. Life-cycle consumption and hyperbolic discount functions. European Economic Review 42 (35), 861 – 871.

Malin, B. A., 2008. Hyperbolic discounting and uniform savings floors. Journal of Public Economics 92 (10/11), 1986–2002.

O'Donoghue, T., Rabin, M., 2003. Studying optimal paternalism, illustrated by a model of sin taxes. American Economic Review 93 (2), 186–191.

O'Donoghue, T., Rabin, M., 2006. Optimal sin taxes. Journal of Public Economics 90 (1011), 1825 – 1849.

Phelps, E. S., Pollak, R. A., 1968. On second best national saving and game equilibrium growth. Review of Economic Studies 35 (2), 185–199.

Salanié, F., Treich, N., 2006. Over-savings and hyperbolic discounting. European Economic Review 50, 1557–1570.

Strotz, R. H., 1955. Myopia and inconsistency in dynamic utility maximization. Review of Economic Studies 23 (3), 165–180.

Thaler, R. H., Benartzi, S., 2004. Save more tomorrow$^{\text{TM}}$: Using behavioral economics to increase employee saving. Journal of Political Economy 112 (1), S164–S187.

Thaler, R. H., Sunstein, C. R., 2003. Libertarian paternalism. American Economic Review 93 (2), 175–179.

Vieille, N., Weibull, J. W., 2009. Multiple solutions under quasi-exponential discounting. Economic Theory 39 (3), 513–526.