Teachers' Conceptions of Mathematics and their Instructional Practices

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Introduction

In this paper will discuss teachers' conceptions about the nature of mathematics and its influences on their teaching practices through the relevant literature. It will particularly refer to Hersh (1986) – focussing on the nature of mathematics, teachers' conceptions of nature of mathematics, and the effects of teachers' conceptions of nature of mathematics on their instructional practice. Closing this paper, it will draw upon my own perceptions and experience of teaching mathematics in relation to Hersh (1986).

Teachers' Conceptions about the Nature of Mathematics

> The issue, then, is not, What is the best way to teach? But, What is mathematics really all about? (Hersh, 1986, p.13)

Mathematics is at the heart of many successful careers and successful lives (National Council of Teachers of Mathematics [NCTM], (1998). Hersh (1986) also defines mathematics as ideas; not marks made with pencils or chalk, not physical triangles or physical sets. Underlying his view of mathematics is that *knowing* mathematics is *making* mathematics (Committee of inquiry into the teaching of mathematics in school, 1983). These ideas may be predicted or constructed by a combination of teachers' beliefs about the subject or the contents. According to Telese (1997), a combination of beliefs may be described as belief system, which is restricted, as individuals reflect on their beliefs. Individual

teachers possess particular beliefs of varying degrees of conviction that develop into personal perspectives of the subject. The belief system is organized in to teachers' conception of mathematics whose components consist of conscious or subconscious beliefs, concepts, meaning, rules, mental images, and preferences concerning the discipline of mathematics (Thompson, 1992). Ernest (1988) believes that the teachers' subject conception resides in their belief system by indicating that the key belief components of the mathematics teacher is the teacher's conception of the nature of mathematics and his or her belief system concerning the nature of mathematics as a whole.

On the significance of the teachers' conception of mathematics, he also argues that although knowledge is important, it is not sufficient by itself to account for the differences between mathematics teachers. For example, two teachers can have similar knowledge: one has the traditional conception of mathematics, emphasizing "…the mastery of symbols and procedures, largely ignoring the processes of mathematics and the fact that mathematical knowledge often emerges from dealing with problem situations" (*Standards* NCTM, 1995, p. 9) and the other has the non-traditional conception of mathematics, emphasizing "the continually expanding field of human creation and invention" (Ernest, 1988, p. 93).

Associated with teachers' conceptions of mathematics are beliefs aligned with the traditional absolutist view and a non-traditional constructivist view of mathematics (Roulet, 1998). Among other views about mathematics, absolutist and constructivist views are distinguished here because of their observed occurrence in the teaching of mathematics (Thompson 1984), as well as in the evidenced teachers' conceptions of mathematics and science (Ernest, 1988).

Teachers' with absolutist conception of mathematics describe the mathematics subject as a vast collection of fixed and infallible concepts and skills (Romberg, 1992) and a useful but unrelated

collection of facts and rules (Ernest, 1989). The teachers adhere to the belief that mathematics is an unrelated collection of facts and mathematical knowledge becomes certain and absolute truths. It represents "the unique realm of certain knowledge" (Ernest, 1991). Finally, Ernest (1996), summarizes teachers' absolutist views about mathematics by saying:

> Absolutist views of mathematics are not concerned to 'describe' mathematics or mathematical knowledge…Thus mathematical knowledge is timeless…it is superhuman…it is pure isolated which happens to be useful because of its universal validity; it is value-free and culture-free, for the same reason. (p. 2)

Another promoted or more "fashionable and fruitful" (Philip, 2000, p. 2) conception of mathematics among teachers is constructivism: "the image of mathematics, which is growing in popularity among mathematics educators" (Roulet, 1998, p. 29). Even the reforms proposed by both the NCTM (1989) and The Ontario Association for Mathematics Education [OAME] (1993) are rooted in constructivism and they support the transition of teachers' mathematics conceptions from the traditional absolutist view to a non-traditional constructivist view (Roulet, 1998). Constructivism is one alternative view to traditional instruction that NCTM promotes (Sandhotz, Ringstaff, and Dwyer 1997; Brook and Brooks 1993). Furthermore, Hersh (1986, pp. 22-23) lists three main properties of mathematical activity or mathematical knowledge which adhere to constructivist view of mathematics and challenge the basic assumption that mathematical knowledge is infallible. These properties are:

1. Mathematical objects are invented or created by humans.

2. They are created, not arbitrarily, but arise from activity with already existing mathematical objects, and from the needs of science and daily life.

3. Once created, mathematical objects have properties that are well-determined, and we may have great difficulty discovering, yet they are possessed independently of our knowledge of them.

From these properties, it seems Hersh advocates the idea of practical mathematics and challenges the assumption that mathematics is absolute and certain.

The constructivist view emphasizes the practice of mathematics and the reconstruction of mathematical knowledge. Teachers holding the constructivist view of mathematics take the subject as a language developed by humans to describe their observations of the world. The teachers see mathematics as continually growing, changing and being revised, as solutions to new problems are explored by the learners with the teachers as "facilitators".

Mathematics teachers may not be able to describe their personal conceptions of the subject in terms of absolutist or constructivist view of mathematics. However, the importance for teaching of such views of subject matter has been noted both across a range of subjects, and for mathematics in particular (Thom, 1973).

The Impacts of Teachers' Conceptions of Mathematics on their Instructional Practices

> A person's understanding of the nature of mathematics predicates that person's view of how teaching should take place in the classroom. (Hersh, 1986, p. 13)

Research has recently begun to emerge indicating that mathematics teachers' conceptions about the subject matter, teaching, and learning influence their action in the classroom(Madison, Nason, and Lanier, 1986; Carpenter, Fennema, and Peterson, 1986; Thompson, 1984; Dougherty, 1990).

Thompson (1992) notes such new areas as the nature of teachers' beliefs about mathematics subject matter and about its teaching and learning, as well as the influence of those beliefs on teachers' classroom practices. In her earlier study, Thompson (1984) also contends that there is a strong reason to believe that in mathematics, teachers' conception (their beliefs, views, and preferences) about the

subject matter and its teaching play an important role in affecting their effectiveness as the primary mediators between the subject and the learners.

Furthermore, according to Ernest (1988), teachers' conceptions of the nature and meaning of mathematics are crucial to teachers' approach to mathematics teaching. However, Hersh (1979) points out the root of the problems in teaching: "controversies about teaching…cannot be resolved without confronting problems about the true nature of mathematics" (p. 33). Thus, Thom (1973) sees the fact the teacher's perception about the nature of mathematics is an integral feature of a mathematics classroom. "In fact, whether one wishes it or not, all mathematical pedagogy, even if scarcely coherent, rests on a philosophy of mathematics." (p. 204)

Teachers' practices are influenced by many factors. Ernest (1988) emphasizes there are these factors (elements): teachers' system of beliefs about mathematics and its teaching and learning; the social context of the teaching situation; and teachers' level of reflection. Although it seems teachers' beliefs about mathematics and its teaching and learning receives attention equal to the other elements, Ernest emphasizes the importance of mathematics teachers' beliefs by claiming that teachers' approaches to mathematics teaching depend basically on their systems of beliefs; in particular on their conceptions of the nature of mathematics, and on their mental models of teaching and learning mathematics.

In spite of some literature suggesting that teachers' conception of mathematics and its teaching and learning are not related in a simple cause-and- effect way to their instructional practices (Pepin, 1999), and, even though there exists some disparities between teachers' conceptions of the subject and their actual practice due to many constrains (e.g., fixed curricula, time pressure and many other external factors) (Reymond, 1993), many researchers and research findings indicate that there is considerable agreement that beliefs influence action (Abelson, 1979). The teachers' subject images in particular,

affect teachers' interpretations of content knowledge (Kitchener, 1986) and their instructional approaches (Pope and Scott, 1984).

The teachers who hold the absolutist view about mathematics and its teaching and learning are more likely: to create teacher-centred instructional environment, teach mathematics as rules to be memorized, and portray mathematics as an infallible discipline. Such teachers who hold this view tend to present mathematics to students in a way that suggests mathematics is a linear subject, facts and skills related to numbers which generally feature paper and-pencil activity. Since the teachers' main objective is the learners mastery of mathematical skills, the clear presentation of the step by step of any mathematical procedure and the emphasis on right or wrong answers are likely to be practiced.

Teachers holding constructivist view of mathematics are expected to adopt teacher-student interaction mode of instruction by allowing students to explore and investigate while teachers reside in their classrooms as facilitators. Problem solving is central to teaching for constructivist mathematics teachers where purposeful activity stems from problem situations that require reasoning and creative thinking, gathering and applying information, discovering, inventing, communicating and testing ideas (Thompson, 1992). Consequently, the classroom takes on a constructivist environment.

Research studies and findings (Pepin, 1999; Teo, 1997) provide evidence that teachers' instructional practices, especially in mathematics, do reflect the teachers' conception of the subject matter. Pepin (1999) studied the conceptions and works of mathematics teachers in three countries: England, France and Germany. The study explores the issues concerning conceptions of mathematics, conceptions of mathematics teaching and learning, and the way in which mathematics teachers' classroom practices reflect teachers' conception of mathematics and its teaching and learning. Pepin's findings suggest that teachers' conceptions are manifested in their practices and can be traced back to

the educational trends of mathematics and mathematics education, as well as to personal constructions. The findings also suggest that teachers' pedagogical style are a personal response to a set of assumptions about the subject (mathematics) and its teaching and learning, to a set of educational and philosophical traditions, and to a set of institutional and societal constraints. Particularly, in case of French and German teachers and their perception of mathematics, Pepin notes (1999, p. 140) notes: "the teachers felt that logic was the principal element of mathematics, and their classroom practice reflected these beliefs." On the other hand, Pepin explains that in England, although teachers talked bout logic and proof as their conceptions of mathematics, their teaching did not include theses aspects and they were concerned with covering the content of the curriculum. Despite this disparity, Pepin concludes her study by stating:

> The findings of the research demonstrate that teachers' classroom practices in the three countries reflected their beliefs and conception of mathematics and its teaching and learning. (p. 144)

Teo (1997) takes a different approach in his research about teachers' beliefs and their instructional practices by conducting a small scale survey of sixteen mathematics teachers in Singapore. Unlike many other studies, Teo did not use the observation method to study the effect of teachers' beliefs on their teaching practices. Instead, he sought the answers about the effectiveness of the teachers' beliefs on their teaching practices from the teachers themselves. Some of his survey questions draw upon findings on teachers' consciousness of their own beliefs about mathematics and if their beliefs influence their instructional practices. Surprisingly, the results of the study indicate that all the teachers (except one) in the survey declared their awareness of their own beliefs about mathematics and felt that these beliefs do influence their instructional practice.

Other researchers have examined how teachers' beliefs shape their practices. Thompson (1984) reported a study in which she researched the relationship of teachers, conceptions of mathematics and mathematics teaching to instructional practice. She demonstrated that there was consistency between the teachers' professed conceptions of mathematics and the manner in which they presented the content to their classes. She also found that teachers posses conceptions about teaching that are general rather than being specific to the teaching of mathematics.

Putnam, Heaton, Prawat, and Remillard (1992) conducted a series of case studies into teaching mathematics for understanding. Results from the studies showed that teachers' beliefs and knowledge affect how they perceive and act upon different messages about changing the way they teach mathematics. The conceptions teachers assumed about what mathematics is and how it is learned had influence on their decisions about what and how to teach. In their case studies, four 5th-grade teachers with absolutist views of mathematics were expected to implement a reformed mathematics (designed to reflect the NCTM *Standards*, 1989) program with only a new textbook for guidance. In addition, Putnam, et al. observed that:

> The teachers in our cases believe that the computational algorithms that pervade the traditional elementary school curriculum constitute the core of mathematics. The teachers have differing views on what it means to understand those algorithms and how important that understanding is, but it is the algorithms of arithmetic that define their mathematics. (p. 223)

For teachers with absolutist views, mathematics is a vast collection of fixed and unrelated collection of facts and rules that have proven to be useful. When faced with the new textbook with the expectation to encourage students to develop what NCTM (1989) calls "mathematical power" and defines the term as “the ability to conjecture, explore, and reason logically to communicate about and

through mathematics; to solve nonroutine problems; and to connect ideas within and between mathematics and other intellectual entities" (p.1), these teachers made major adjustments to the reformed program to regulate teaching practices compatible with their beliefs. Putnam (1992) writes:

> In the textbook lessons that she did follow closely, Valerie [Putnam's assigned pseudonym] highlighted procedural aspects of content and downplayed opportunities for students to reflect on and discuss mathematical ideas...She spend the lesson time carrying out the computational steps for getting [the result]. (p. 176)

The cited research literature and research findings discussed above provide strong evidence to suggest that teachers' conceptions of mathematics have effect on their teaching practice in a number of ways.

Conclusion

In conclusion, I find Hersh's message embodied in the quotation below fitting with my professional practice as a classroom mathematics teacher:

> One's conception of what mathematics is affects one's conception of how it should be presented. One's manner of presenting it is an indication of what one believes to be most essential in it... The issue, then, it is not, What is the best way to teach? But, What is mathematics really about? (p. 13)

I interpret this quotation to signify what teachers consider to be more effective ways to teach mathematics as dependent on their beliefs and conceptions about mathematics. In view of this, I will present my beliefs about mathematics, mathematics teaching and learning, linking it to my personal teaching experiences, which reflect my conception of mathematics.

Cooney (1994) has stated that mathematics teachers' beliefs about mathematics, mathematics teaching and learning have been shown to critically influence what happens in the classroom. In order to

position myself within the mathematics education conceptual paradigm, it is necessary to present my own conception of mathematics, mathematics teaching and learning as being aligned within the non-traditional constructivist view about mathematics, teaching and learning mathematics. I believe mathematics is a man-made universe to construct and revise our knowledge. My belief about mathematics teaching and learning is centered on the adoption of teacher-student interaction mode of instruction by allowing students to explore and investigate, while I reside as a facilitator in my classroom. Associated with my teaching practice, problem solving, discovery, group working and creativity are central to my teaching strategies. As a teacher, I attempt to enhance the conceptual and practical understanding of mathematical problems through integration of subjects, especially in non-classroom settings. For example, taking students outside on a sunny day and asking them to measure the height of a tall building (without climbing it) or the width of a river (without crossing it) to show the integration of geometry and mathematics, as well as the practical side of mathematics. One of the strategies that I advocate and use to show my students the practical side of mathematics is revealing the specialized mathematics involved in many professions such as tiling (applications of rational and irrational numbers) in construction, map colouring (application of graph theory) in Geo-Centres.

Acknowledging the existence of many flaws in constructing a constructivist environment in my classroom, I consider the desirable way to teach mathematics is through constructivist paradigm which rests, to a large extent, on my belief and conception of mathematics.

Also, admitting my awareness of the influence of my conception of mathematics on my instructional practice of the subject may be assumed as another evidence to Hersh's (1986) and Cooney's (1994) statements on teacher's conception of subject matter and its effects on teaching practice.

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