AN ANALYSIS OF VIEWS OF THE NATURE OF MATHEMATICS BY GENDER

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INTRODUCTION

The purpose of this enquiry is to examine, by gender, pupils' perceptions of the nature of mathematics.

This has been attempted by administering a survey in which the questions fall under two categories, dualist and relativist, which are shown to correspond to a greater or lesser extent to absolutist and fallibilist philosophies respectively. Dualism and relativism are discussed in detail both in connection with school mathematics and mathematicians' mathematics. Some hypotheses are put forward about the sources of the driving forces behind the new mathematics' curricula. In the same vein, the fact that the rise in awareness about gender differences took place just after these curriculum upheavals is examined for any causality.

The design of the experiment itself is seen as critical, with much attention being given to avoiding leading the pupils to preordained or stereotypical responses. The data is analyzed with boys and girls plotted against dualist and relativist viewpoints. Any gender differences, or lack of them, are discussed with reference both to the existing literature and to the design of the experiment itself.

LITERATURE REVIEW

During the past thirty years many studies have been done on gender differences as they relate to achievement and participation in mathematics. Differing attitudes to, and perceptions of, the subject itself have also been researched.

However, before pupils' perceptions of the subject of mathematics can be analysed, the nature of mathematics itself must be examined, Interestingly, there may not be just one mathematics. "Lerman (1983) identified two alternative conceptions of the nature of mathematics, which he called absolutist and fallibilist views." (Grouws, 1992, page 132). Also "Copes (1979).... proposed four types of conceptions: absolutism, multiplism, relativism, and dynamism." (Grouws, 1992, page 133) while William G. Perry Jr. in the 1980's and 1960's had gone so far as to identify "nine stages or 'positions' that describe the intellectual and ethical development of college students from the viewpoint of their conception of knowledge." (Grouws, 1992, page 132). For the purposes of this paper, and in harmony with the design of my particular experiment, the absolutist and fallibilist views of pupils are analysed.

The absolutist view of mathematics is of a body of knowledge whose "truths appear to everyone to be necessary and certain" (Ayer, in Ernest, l991, page 7). The whole rests on certain assumptions which are held to be self-evident, Indeed, strict absolutists hold that mathematics is almost independent of humankind, existing as it does in its government of nature, binding the Universe together with its unfailing consistency across time and space. This consistency has been, to the absolutists, one of mathematics' most powerful appeals. Who can resist the simple demonstration of an apple being dropped repeatedly and taking the same amount of time to hit the ground in every case? Little wonder that for several thousand years the nature of mathematics went largely unquestioned. Axioms provided the basis for all mathematical knowledge. On these were built, using logic, the theorems and proofs which constituted the subject itself. Mathematics was widely regarded as value-free, at least partly because the laws of nature were not dependent upon the presence of humankind. However, statements such as the following from late Victorian times: "All secondary schools.....insofar as they qualify *men* for doing something in life, partake more or lese in the character of institutes that educate craftsmen." (1895 Bryce Report on Secondary Education, in Ernest, 1991, page 156, emphasis added) actually reveal a significant number of values, at least in mathematics pedagogy. Here mathematics was being viewed by an early technological pragmatist whose belief system, by definition, did not allow for the questioning of basic mathematical principles or the means by which they were derived. In a more up-to-date setting, two other educational ideologies may be grouped under the banner of absolutists. These are those of the industrial trainer and the old humanists. Implicit in, and fairly central to, their pedagogical styles, is the role of the teacher as authoritarian and the pupil as empty vessel, the fallibilist view has been developed more recently and may be seen to be partly a reaction to absolutism in general - "It is now apparent that the concept of a universally accepted, infallible body of reasoning.....is a grand illusion." (Kline, in Ernest, 1991, page 18) – and partly a new wave of thought in its own right. The roots of this philosophy may spring from the realisation that "there are no authoritative sources of knowledge, and no 'source' is particularly reliable.......we are all fallible." (Popper, in Ernest, 1991, page 15). Immediately the contrast with the absolutist view can be seen. The fallibilists regard mathematics as an essentially human pursuit, invented by humans, and therefore prey to human fallibility. They do not subscribe to a belief system based on incontrovertible truths in the unconditional way that absolutists do. Instead, fallibilists argue "Why not honestly admit mathematical fallibility......rather than delude ourselves that we shall be able to mend invisibly the latest tear in the fabric of our 'ultimate' intuitions." (Lakatos, in Ernest, 1991, page 19). Under the heading of fallibilists we shall group both the progressive educators and the public educators.

Both of these philosophies, the absolutist and the fallibilist, are expressed from the mathematician's standpoint. However there is a 'trickle-down' effect to school mathematics and the way it is presented: "Different philosophies of mathematics have widely differing outcomes in terms of educational practice." (Ernest, 1991, page 111). It is perhaps no coincidence that this crisis of faith' in the absolutist methods occurred just before the introduction of the 'new maths in schools. "Crowing concern over declining enrollments in university mathematics courses were beginning to give rise to a flood of curriculum reform projects in various countries that collectively became known as 'the new math'" (Grouws, 1992, page 23). Educators were eager to present mathematics without the constant explicit references to axioms and proofs, perhaps for the simple reason of increased accessibility or because of the deeper epistemological changes taking place in the views of the academics. For instance, by bringing human intuition into school geometry and allowing proof by methods other than by reference to certain axioms, mathematics educators were mirroring, either consciously or unconsciously, the changes taking place on the way mathematics was being viewed by philosophers.

As soon as the climate was right for curricular change, any relationships between gender, course content, and pedagogy could be fed into the educational model as a whole. Research began in earnest into gender bias not only because perceptions of the nature of mathematics were changing, but also because the consciousness of women was being visibly raised at around the same time. Research into participation rates by gender in higher level courses has even gone outside the curricular and pedagogical domains: “The success of mathematics in attracting middle class males is more apparent than real, being more closely related to its prestige and its status as a ‘critical filter' between school and employment than to any fundamental mathematical qualities of the courses offered” (Russell, in Willis, 1989, page 32). Although many researchers have examined the issues of whether or not mathematics is a subject more suited to boys than girls and whether or not boys find maths easier than do girls, the social forces at play cannot be underestimated in their influence in University choices – "while in theory U.K. schools offer equality, they also engage in a process of ‘cooling out' aspirations in girls in preparation for a long-term future of inequality." (Boswell, in Willis, 1989, page 32).

THE EXPERIMENT

The purpose of the experiment was to obtain a hopefully unbiased record of the attitudes of boys and girls towards mathematics in order that these attitudes might be analysed, contrasted and compared. The questionnaire used was based on the work of William Perry as discussed in Ernest (1994, page 31). Using the questionnaire, the dualist and relativist views of mathematics among pupils were elicited.

The experimental design was formulated in order to meet a number of criteria. A well-designed questionnaire should effectively disguise its underlying purpose, ensuring that the respondents will not artificially generate responses in order to fit their answers into what they believe is a pattern desired by the interviewer. In order to achieve this, the mathematics items were balanced with items concerning attitudes to the subject of English. Further, the questionnaires were administered by a health education teacher to avoid a perceived bias towards either the mathematics or English subject area. As instructed, the health education teacher presented the questionnaire with the simple introduction of, "This is a general school survey. Answer to the best of your ability.” The final and perhaps more important blind consisted of the omission of a single bracket in the top right-hand corner of the girls’ questionnaires (in every other respect the boys’ and girls’ questionnaires were indistinguishable) which allowed both the distribution and collection of the questionnaires to be gender specific without the knowledge of the pupils. The papers were distributed from a single pile with the girls' papers being dealt from the top and the boys' papers from the bottom. The random collection of the papers was critical, as the three sub-groups of the whole year group in the experiment were held over two days: any analysis of the purpose or procedures by the pupils had to be avoided if at all possible.

The entire third year (approx. 14 yrs of age) was used in the experiment: 42 girls and 32 boys. They attend a selective, academic, co-educational secondary school whose population comprises approximately the 'top' 40% of the general population as identified by a total of two hours of mathematics and English entrance examinations. This method of selection combines the mathematics and English scores and so allows for a slightly wider range of abilities in the mathematics than simply the 'top' 40%. The children are from all socioeconomic backgrounds and nearly all have lived in Bermuda since birth. The first language for the entire school is English. The decision to target a single year group was made in order prevent age from being a factor in the analysis of the data.

The mathematics items in the questionnaire were taken from an existing one in the course booklet "The Nature of Mathematics and Equal Opportunities" (Ernest, 1994, pages 31-32), as were the distracting items on attitudes to English. All of the mathematics responses were scored on an ordinary scale with Strongly Agree given 5, Agree 4, Undecided 3, Disagree 2, and Strongly Disagree 1. With every dualist question being phrased so that an agreement with it gets a high score, this enabled all the scores within the dualism classification to be added together and averaged. Thus a high average on the dualistic questions indicated agreement with the dualistic perspective. Relativist items were scored in the same way. A spreadsheet of raw scores (included in the appendix – omitted here) was produced and the following four averages were calculated:

	Dualist	**Relativist**
Boys	2.91	3.67
Girls	2.91	3.48

It is clear from these statistics that there is little gender difference in views in terms of dualism and relativism.

CONCLUSION

Three interpretations of the results may be possible. First, there are indeed no gender differences in the way these pupils view mathematics. Second, there are gender differences, but this experiment did not pinpoint them. Third, any gender differences are slight, but masked or outweighed by other factors.

If there are no gender differences there would have to be good reasons, as this would appear to contradict the findings of Preston (1975) who "found that significantly more girls ....tended to see mathematics as an algorithmic, mechanical and somewhat stereotyped subject. Boys tended to see mathematics in an open ended, intuitive and heuristic setting." (quoted in Ernest, 1994, page 32). One immediate and important difference between this experiment and Preston's may well account for this contradiction; namely that twenty years separates them. In the intervening time, teachers may have become more sensitive to any perceived gender biases because of a wide range of studies such as that done by the American Association of University Women reported in their book: "How Schools Shortchange Girls: Action Guide. Strategies for Improving Gender Equity in Schools" which may have had an impact on teacher training college syllabi. Also, textbooks have had gender biases in the past – Dowling (1991) found examples of gender bias in the texts of the School Mathematics Project series – but there is evidence of change over time, e.g. with reference to the SPMG Primary Maths Scheme – "In the sample from the 1978 edition.....an overwhelming 81% of the pictures of persons are male.....In the 1988 edition.....very nearly equal numbers of male and female images were shown."(Ernest, l994, page 45). Lack of gender differences found in this experiment could therefore be explained by changing teacher attitudes in conjunction with less biased teaching materials.

However, great care must be taken here to point out that the many published studies concerned with gender differences in attitudes to mathematics deal with a variety of issues within this broad topic; such as how much pupils view their own rank within a group. Mura (1987) found that "While for both sexes more students overestimated than under estimated their performance, males were more likely than females to overestimate, females more likely than males to underestimate, their final grades." (quoted in Grouws, 1992 page 604). Another area of attitudinal study concerns to what pupils attribute their own particular success or failure. Wolleat, Pedro, Beaker, and Fennema (1980) found that "Females, more strongly than males, attributed success and failure in mathematics according to a pattern described as learned helplessness. Females were more likely to use effort but less likely to give ability as explanation for success; ability and task were more often perceived as the reasons for failure." (quoted in Grouws, 1992, page 601). Additionally, "much of the research on girls and mathematics talks of the problems caused by the stereotypical perceptions of mathematics as a male domain" (Ernest, f994, page 31). As this experiment dealt with dualist and relativist attitudes, nothing wider, it might be fallacious to imply that exposure to literature on gender bias in general attitudes to mathematics would be enough to sensitize teachers to such an extent that their pupils did not view mathematics in a stereotyped way according to gender.

As was mentioned before a second possibility exists: there are gender differences but the experiment failed to highlight them. There are several possible explanations as to why this occurred.

There may have been too few questions, especially those trying to elicit relativist attitudes. An attempt was made to rectify this situation even before the experiment was carried out. In the original questionnaire (Ernest, 1994, page 31) there were five relativist questions. The original question number 4 "Some maths problems have many answers, some have none" was spilt naturally in two to form question numbers 8 & 12 in this experiment. This added another relativist item without destroying the integrity of the original questionnaire. However, in order to fully overcome the design flaw of too few questions, more questions would have to be added. A possible side effect of adding questions might be that comparisons between the results from the longer and shorter questionnaires could prove be less reliable.

Suspect question content may have contributed to the insensitivity of the questionnaire to actual gender differences in attitudes towards mathematics. For instance, question number 22 ”There is only one correct way to solve a maths problem" may be so dogmatic that its wording contributed in no small way to its ratings of 1.53 and 1.90 on the 1 – 5 scale, i.e. a large number of pupils, be they dualist or relativist, strongly disagreed with the statement. Question number 14 “Maths is basically doing sums." may be both obviously simplistic and inappropriate for the sample age group as most of their coursework concerns problem solving. This could explain the ratings of 2.06 and 2.10 for the boys and girls respectively.

Other studies have had many more respondents, such as that of Preston, who had ”almost three thousand school secondary children" (Ernest, 1991, page 32). As sample size increases, a truer statistical picture of the situation will emerge. In a small experiment such as this, the responses of three or four individuals can have a profound effect. Few studies can be harmed by an increase in the sample size.

"This questionnaire is based on a *simplified* dichotomy of views of the nature of mathematics" (Ernest, 1994, page 31, emphasis added). Perhaps this categorization is too simple, in that it does not allow for the full range of perceptions of maths which might exist in the pupils questioned. As mentioned earlier in this paper, there are other, more detailed ways of categorizing views of the nature of mathematics.

The third possibility might be explained with reference to the teachers and to the pupils. The homogeneity of the mathematics courses taken by this group of pupils may have been a big factor. In this sample, all of the respondents have been taught by only four (male) teachers in the last two and a half years. These teachers have worked together closely, and in doing so may have taught the material in such a way that mathematics considerations (such as investigational work) have been so strong that they have outweighed, or masked, any real gender differences in attitudes. Also, the group of pupils itself is not representative of the general population. As was mentioned earlier, this group comprises the ’top' 40% of the population

as measured by their achievement in an 11+ type of test. Perhaps the pupils, being part of such a segment of the population, will tend to have fairly similar views, by gender, of the nature of mathematics. This may not be an insupportable assumption. Pupils who have exhibited high achievement in mathematics, it could be argued, may have done so partly because of their similar views on the nature of mathematics, be they male or female. Again, some masking of gender differences may have taken place, but in this instance because of the homogeneity of the pupil sample itself.

One suggestion for possible further research is a longitudinal study. Ideally this investigation would incorporate the design improvements described earlier in this paper, using as a population sample, one year group of pupils surveyed in their first, third, and fifth years in secondary school. Any changes over time in the views of the nature of mathematics could be examined. Perhaps children's views are fixed by a certain age, change as children mature, or are influenced by other factors such as having male or female teachers. The answers to these questions could have important implications for teaching methods and syllabus design.

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