PRAGMATIC PERSPECTIVES ON MATHEMATICS DISCOURSE

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Within the psychological tradition, analysis of students' utterances has focused on the 'transactional' function of language in the expression of 'factual' propositional content. The clinical interview is an effective means of eliciting data with this (though not only this) objective (Rowland, 1999a, b). The purpose of this paper is to emphasise that language also fulfils an essential 'interactional' function in expressing social relations and propositional attitude, and to argue for the significance of such dimensions in the analysis of discourses of a mathematical nature. The paper explicates the nature of 'pragmatic'[1] meaning, reviews some approaches to discourse rooted in or related to speech act theory (Austin, 1962; Searle, 1969), and concludes with analysis of a fragment of text for illustrative purposes.

MEANING

There exist a number of accounts of 'meaning' within natural language. Accounts differ partly because of different perspectives deriving from linguistics, philosophy, psychology, sociology, literary criticism, theology, and so on. In discussing meaning in this paper, I have in mind a view of communication that focuses on human intention, and my starting point is what the philosopher Paul Grice (1957) called 'speaker meaning'. Grice distinguishes between two kinds of meaning: natural and non-natural. The first of these might be called the semantic or truth-conditional meaning of sentences such as:

if $x>2$ then $x^2>4$

This is meaning of a conventional, literal kind. Of course, such a notion is not unproblematic, pre-supposing as it does that words and symbols refer to things in an unambiguous way, and that the syntax of the sentence then takes care of the meaning.

Non-natural meaning is associated with (human) communication which is intended to be received in a particular way by a recipient. This is the 'pragmatic' meaning, which *may* be conventional, but certainly need not be in the case, for example, of ironic, metaphorical and indirect communications. A critical feature of non-natural meaning is the particular way that it is *intended* to be recognised and interpreted by a recipient. The distinction between the truth-conditional, semantic meaning and the pragmatic meaning of an utterance is demonstrated by B's turn in the following interchange.

A: Do you think you'll get back home in time to cut the grass?

B: Well, I'll try, but there are road works on the A14 this week.

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[1] It should soon become clear that 'pragmatic' is used here, and throughout in this paper, in a technical sense, so that it means something more specific than just 'expedient' or 'dogma-free'.

The semantic content of the reply is to the effect that B will endeavour to get home on time, and that there are works on a particular road. In practice, we read much more into it. Even in the absence of 'well', we might infer *inter alia* that:

- B will travel by road;
- the road works on the A14 might cause B to be delayed;
- B is not confident that s/he will be home in time to cut the grass.

The inclusion of 'well' adds an additional dimension to the reply: that is, B's anticipation that his or her reply is not the one that A would like to hear (Wierzbicka, 1976; Brockway, 1981).

In an account of the 'transactional' and 'interactional' functions[2] of language, Brown and Yule touch on a distinction which seems to parallel that between semantic and pragmatic meaning.

> That function which language serves in the expression of content we describe as *transactional*, and that function involved in expressing social relations we will describe as *interactional*.
>
> Whereas linguists, philosophers of language and psycholinguists have, in general, paid attention to the use of language for the transmission of 'factual propositional information', sociologists and sociolinguists have been particularly concerned with the use of language to negotiate role-relationships, peer-solidarity, the exchange of turns in a conversation, the saving of face of both speaker and hearer. (Brown and Yule 1983, pp. 1-4)[3]

The importance of the transactional function of language in the teaching and learning of mathematics is self-evident. Michael Halliday, a linguist, leaves us in no doubt as to the educational significance of the interactional function.

> If we consider the language of a child, there is good evidence to suggest that control over language in its interpersonal function is as crucial to educational success as its control over the expression of content, for it is through this function that the child learns to participate, as an individual, and to express and develop his own personality and his own uniqueness. (Halliday, 1976, pp. 197-8)

Pragmatic meaning is the means frequently (though not necessarily consciously) used by speakers to convey affective messages to do with social relations, attitudes and beliefs, or to associate or distance themselves from the propositions they articulate. That is to say, pragmatic meaning is an important tool in fulfilling the interactional function of language.

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[2] This binary subdivision of the functions of language is adequate for our present purposes. A finer taxonomy of speech functions due to Roman Jakobson (1960) identifies six functions: referential, emotive, conative, phatic, metalingual and poetic.

[3] Grice and Austin demonstrate that philosophers of language are by no means exclusively preoccupied with transaction.

Shiffrin (1994) and Jaworski and Coupland (1999) give excellent surveys of a wide range of approaches to discourse which are particularly sensitive to pragmatic meaning and interactional function. The following sections of this paper give a necessarily brief overview of three contributions to pragmatic analysis. Each points to possible means whereby non-natural meaning might be conveyed by a speaker, and each will play a part in the scrutiny of a teaching episode towards the end of this paper.

SPEECH ACTS

In the late 1950s, the Oxford philosopher John Austin gave some lectures on how speakers "do things with words", and so invented a theory of 'speech acts' (Austin, 1962) which now occupies a central place in pragmatics. The essential property of speech acts is that they bring about (or have the potential to bring about) a change in some state of affairs. Paradigm examples include the naming of a ship, the joining of two persons in marriage, and the sentencing of a criminal. The name, the marriage and the sentence are what they are because an authorised person has declared them to be so. Austin distinguished between the *locution* of a speech act (the words uttered), its *illocution* (the intention of the speaker in making the utterance) and its *perlocution* (its effects, intended or otherwise).

Whereas declarative utterances typically have truth conditions, speech acts must satisfy certain 'felicity conditions' in order to 'count' as an action. For example, the felicity conditions for a *question* include the expectation that the enquirer doesn't know the answer, that s/he would like to know it, and has reason to believe the hearer is able to supply it. Questions in classroom situations are curiously exempted from these rules (Labov and Fanshel, 1977).

One aspect of speech act theory with significant pragmatic implications concerns *indirectness*. Three broad illocutionary categories are normally identified - statement, question and command/request - having typical realisations in declarative, interrogative and imperative verb forms. These agreements between intended function and realised form break down in 'indirect speech acts', in which the outward (locutionary) form of an utterance does not correspond with the intended illocutionary force of the speech act which it performs (Levinson, 1983, pp. 263ff). Common forms of this are to declare a preference or to use an interrogative form in order to convey an order or request. For example:

Teacher: I'd like to take in your exercise books.

Diner: Can you bring me the wine list?

These are both instances of how speakers frequently accomplish an indirect speech act by stating or questioning one of the felicity conditions (Gordon and Lakoff, 1971). The teacher explicitly *states* his wish to receive the books i.e. that s/he meets the felicity condition to do with speaker sincerity; the diner *questions* the ability of the waiter to provide the list i.e. s/he questions one of the preparatory pre-conditions.

POLITENESS

So why should speakers in general, and teachers in particular, be indirect in this way?

One insight into such indirectness in classroom mathematics talk is provided by a sociolinguistic theory of 'politeness' developed in the late 1970s. This theory claims that speakers avoid threats to the 'face' of those they address by various forms of indirectness, vagueness, and so on, and thereby 'implicate' (hint at) their meanings rather than assert them directly. Politeness theory (Brown and Levinson, 1987) is based on the notion that participants are rational beings, but with two kinds of 'face wants' connected with their public self-image:

- positive face - a desire to be appreciated and valued by others; desire for approval;
- negative face - concern for certain personal rights and freedoms, such as autonomy to choose actions, claims on territory, and so on; desire to be unimpeded.

Now some acts ('face threatening acts', or FTAs) intrinsically threaten face. Orders and requests, for example, threaten negative face, whereas criticism and disagreement threaten positive face. The perpetrator therefore must either avoid such acts altogether (which may be impossible for a host of reasons, including concern for her/his own face) or find ways of performing them whilst mitigating their FTA effect.

Brown and Levinson identify and catalogue a number of related linguistic strategies, including quasi-interrogative commands (such as that of the Diner, above) which redress the threat to the addressee's negative face, their autonomy, respecting their right to refuse. These include conventionally polite, indirect speech acts such as "Can you pass the salt, please?".

HEDGES

The linguist George Lakoff coined the term 'a hedge' for a word or phrase that makes a proposition "fuzzy" or vague in some way (Lakoff, 1972). A hedge can be categorised either as a 'shield' or as an 'approximator'.

Maybe, probably and *possibly* are examples of so-called 'plausibility shields', which stand outside a substantive proposition (e.g. '[Maybe] we should call a taxi') and point to something less than complete commitment to it. One of the functions of shield-hedges is to protect the speaker from accusation of being committed to a false proposition (Channell, 1994). Incidentally, teachers more often use 'attribution shields' such as "so-and-so says that …", distancing themselves from a proposition by attributing it to someone else. This is a pedagogical strategy which avoids 'closing' on a problem, in order to sustain discussion and invite a variety of proposals. For example:

Teacher: John says you can't divide 739 by 9. What do other people think?

Approximators such as *about, around* and *approximately*, as well as *sort of, kind of* and *basically*, can, like shields. also have the effect of withholding commitment to a proposition. They achieve this by inserting vagueness into the substantive proposition itself (e.g. 'The taxi will be here in [about] ten minutes').

Here, a 14-year-old boy makes a prediction, but the vagueness of his answer suggests that it was far from secure:

Allan: The maximum will probably be, er, the least'll probably be 'bout fifteen.

Allan hedges his prediction in two ways : the shield 'probably' is reinforced with the approximator '(a)bout'. The very act of complying with the teacher's request for a prediction is a threat to Allan's positive face, since he could be thought foolish if his prediction were subsequently found to be in error. The shield makes his lack of commitment explicit; use of the approximator 'about' is a more subtle protective strategy, for it renders Allan's answer "almost unfalsifiable" (Sadock, 1977, p. 437).

There follows a fragment from one mathematics lesson. The discussion of the interaction here focuses on the pragmatic meanings of some of the utterances within the text, with concern for the beliefs and attitudes of the three participants towards the subject-matter and each other. The choice of this fragment is to some extent determined by the 'conjecturing atmosphere' (Mason, 1988, p. 9) that permeates the episode. It is this factor, I believe, that makes the 'conversation' below a special kind of discourse. As I have argued in Rowland (1999b), such an atmosphere involves the student (and possibly the teacher) in taking risks, in articulating generalisations on the basis of partial evidence. Evidence of the management of such risks, by teacher and student, is to be found in pragmatic discourse analysis. The discussion which follows also involves reference to aspects of justification and proof which have not been previewed in this introduction, but will be familiar to researchers in mathematics education.

DISCOURSE ANALYSIS: HAZEL

Hazel, an elementary school teacher, describes 10-year-old Faye and Donna as able mathematicians who often work together. Her discussion with them is essentially an exploration of the following problem.

> Take three equally spaced numbers, such as 10, 13, 16. Find the product of the outer pair [10x16=160] and the square of the middle term [13x13=169]. The difference is 9. What will happen if you take other similar number-triples? What if you take a common difference other than 3?

Hazel's conversation with the two girls falls into four episodes, the first of which is the main focus for this analysis:

> Episode 1: Investigation of the case when the common difference is 1 [turns 1-61]
>
> Episode 2: Investigation of the case when the common difference is 2 [62-105]
>
> Episode 3: Investigation of the case when the common difference is 3 [106-120]
>
> Episode 4: Search for a higher-level generalisation which includes the three generalisations arrived at inductively in the previous episodes as special cases [121-160].

In every case Hazel's instructions and requests to the two girls are presented as indirect speech acts, for example (there are many):

17 Hazel: Shall we try it out and see what happens? Do you want to each choose your own set of consecutive numbers?

66 Hazel: Right would you like to try out with ten, twelve and fourteen one of you and the other one can try another jump.

130 Hazel: Can you tell me what the difference in the answers of the two sums that, the two multiplications you're doing would be when you have a difference of four between each number?

17 and 66 are on-record FTAs, 'orders' presented as questions out of respect for the children's negative face, as Hazel imposes on their personal autonomy of action. These are conventionally indirect. She believes that the investigation will be a worthwhile, educative experience for them with a potentially stimulating outcome. Nonetheless she recognises the risk-taking which is inherent in her quasi-empirical approach, and that she requires their cooperation as active participants in the project as they generate confirming instances of generalisations-to-come. In [17] she says "Shall we try it out?", the plural form including and identifying herself as a partner in the enterprise. In [130] she probes for a prediction, and realises the threat to the girls' positive face - what if they fail to make a correct prediction, will their reputation as "good mathematicians" be dented? [130] respects their positive face, and the indirect modal form redresses the on-record FTA. These features of Hazel's language are manifestations of her 'sensitivity to students' (Jaworski, 1994). Fallibilistic teaching, inviting conjectures and the associated intellectual risks is unimaginable if the teacher is not aware of the FTAs that are likely to be woven into her/his questions and 'invitations' to active participation. Redressive action dulls the sharp edge of the interactive demands that this style places on the learner. For Hazel, notwithstanding her authority in her own classroom, the indirect speech act has become a pedagogic habit.

Early in the conversation, Faye [9] observes a difference of 1 between 10x12 and 11^2. Somewhat precipitately, perhaps, Hazel asks:

10 Hazel: One number difference ... do you think that will always happen when we do this ... ?

Faye readily agrees, but Hazel, perhaps realising that she has not probed but has 'led the witness' seems to want to give them more of an option to disagree.

12 Hazel: What makes you think that? Just 'cos I asked it ... or ...?

Donna gives hedged agreement [14]: Hazel invites her [15] to account for her provisional belief.

14 Donna: I think so.

15 Hazel: Why?

Arguably this is a tough question - to account for a belief that one is not really committed to anyway. Donna's justification [16] is phenomenological rather than structural.

16 Donna: Well if um ... if it's after each other like ten, eleven, twelve ... um ... it will be one more because it's one more going up.

It is the basis of a subsequent higher-level generalisation at the beginning of Episode 2.

62 Hazel: Okay. Right, what would happen if you had numbers that jumped up in two instead of one, so you had ten, twelve and fourteen?

63 Faye: I think the answer is a two number difference. So two.

64 Donna: Yeah, yeah. So do I.

The substantive proposition in [63] - that there is a two number difference - is, in fact, false. By prefacing it with a Shield, Faye marks her utterance as a conjecture, withholding commitment to it.

Returning to Episode 1: Hazel encourages the children to try out two more examples with three consecutive integers. They obtain a difference of 1 in each case and Faye [27] affirms her belief (unhedged) that, as Hazel puts it [26], "that will always happen".

26 Hazel: Do you think that will always happen then?

27 Faye: Yes.

28 Hazel: How can you say for certain 'cos you've only tried out three examples?

Donna offers a brief diversion:

35 Donna: I don't think it will happen if you do like eleven, fourteen, twenty-two.

36 Hazel: But you're talking about the one that ... if you always have a set of three consecutive numbers will it work?

Her "like eleven, fourteen, twenty-two" is a delightful example of a vague generality; what like-ness does she intend to point to with this single example? It is difficult to judge how Hazel interprets it, except that she takes it to exclude "three consecutive numbers" - and perhaps this is precisely what Donna intended to convey through her example. Evidently 'consecutive' is a useful but neglected item in the mathematical lexicon.

Faye brings the discussion back on course with a request for a what philosophers of science might call a 'crucial experiment' (Balacheff, 1988) - testing the conjecture with an example well outside the range so far considered, to explore the extent of its validity.

38 Faye: I'd like to try it out in the hundreds.

Donna's choice for the experiment seems to be guided by Hazel:

39 Hazel: [to Donna] You want one difference between each of those. If you're going to start with a hundred you could have a hundred and one, a hundred and one and a hundred and two. Would you like a calculator ...?

Faye makes an independent choice [60] of 110, 111, 112:

51 Faye: I still get one number different.

52 Hazel: So that ... so do you ... will it always work d'you think?

53 Faye: Yeah ... I think.

54 Hazel: How can you be sure?

55 Donna: Umm

56 Faye: [laughing] Well ...

57 Hazel: Are you sure?

58 Faye: Well not really, but ...

59 Donna: Quite yeah.

60 Faye: I think so. Yeah quite sure. Because it has worked because we've done ten, eleven ... Well I've done ten, eleven, twelve, nine, ten, eleven which are quite similar and then I've jumped to, um, um ... a hundred and ten, a hundred and eleven, and a hundred and twelve. It's quite a big difference. So yeah?

61 Donna: Yeah so do I.

By this stage Hazel seems reluctant [52] to influence their commitment to the generalisation (the 'it' that 'always works'). Faye's intellectual honesty is very evident here. Her crucial experiment [60] provides another (presumably weighty) confirming instance of the generalisation [51] yet her assent to it is still hedged, partial [53]. One senses that Hazel has created, or nurtured, a 'Zone of Conjectural Neutrality' (Rowland, 1999b) in which Faye understands that it is the conjecture ('it always works') which is on trial, not herself. She is free to believe or to doubt. Nevertheless, her 'well's [56, 58] indicate that she senses, perhaps, that it would be easier if she agreed - that agreement would better respect Hazel's positive face wants - for Hazel would gain satisfaction from Faye's coming-to-know.

CONCLUSION

Classroom talk is a rich resource for the analysis of students' cognitive structuring of mathematics, in which student errors are a particularly rich basis for conjectures about fundamental mathematical misconceptions. Such analyses provide essential diagnostic insights into individual knowledge construction. Reports of such analyses are typically set within a framework of knowledge about mathematical cognition. This may be of a general kind, to do with concept formation, abstraction and so on, or related to knowledge about the construction of knowledge in particular topic areas such as fractions or functions.

Concern for more interactional features of the classroom, such as students' propositional attitudes and teachers' sensitivity to their students' self-esteem, necessitates rather a different set of lenses and analytical tools from which to view texts. Linguistic tools which focus on pragmatic meaning have significant potential for text analysis in mathematics education, especially in research into social and affective factors in the teaching and learning of mathematics. Further evidence of this potential is given in Rowland (1999b). A recent paper by Bills (2000) explores the prevalence, purpose and effect of a range of politeness strategies in mathematical dialogue involving a teacher and two 17-year-old students.

Any analysis of classroom interaction involves the selection and application of analytic perspectives, and pragmatic tools are as yet novel in the field of mathematics education. It will be interesting to see what further insights they yield for researchers

as they come into more general use, what the pedagogical application of such insights might be, and whether teachers perceive them as valuable.

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