Engineering and Physical Sciences Research Council Doctoral Landscape Award

PROJECT TITLE: Properties of dynamical systems with an infinite invariant measure and an unbounded observable

Lead Supervisor: Dr Tanja Schindler, University of Exeter - T.Schindler@exeter.ac.uk

Co-Supervisors: Professor Mark Holland, University of Exeter - M.P.Holland@exeter.ac.uk

Webpage: https://tanja-schindler.github.io/

Project details: We consider an ergodic system (X, μ, T) and an observable f:X→ℝ. It is one of the most classical problems in ergodic theory to study the long term behaviour of f,...,f∘T^n; in particular of the Birkhoff sum f+...+f∘T^n. Such systems are often better suited as models than iid random variables as short term dependencies can be taken into account. Besides the easiest case - to study limit theorems for μ a probability measure and f an integrable (often even Hölder continuous) observable - in the last 10 to 20 years a number of limit theorems have been proven for the following settings: a) μ is probability measure and f is a non-integrable observable - in this setting often clustering, i.e. the successive occurance of large events, made those dynamical systems behave qualitatively different from iid random variables with the same distribution function. b) μ is an infinite measure space and f is either integrable or at least bounded - this setting has e.g. been used to model anomalous diffusion.

In this project the general aim is to combine the above two settings, namely we assume that μ is an infinite measure and additionally f is unbounded or even non-integrable over a finite measure set. In this setting, both the infinite measure as well as the non-integrable observable influence the qualitative behaviour and it is the aim of the project to study under which conditions each of the two has the leading influence on the system. Here, depending on the student's progress and interest, three main questions can be considered:

A) The main limit theorems to be proven for such settings are stable laws, extreme value laws, and almost sure limit theorems - possibly two sided and possibly under truncation.

B) A suitable notion of mixing is supposed to be developed. While there are suitable notions of mixing for either of the settings (ψ- or α-mixing in setting a) and Krickeberg-mixing for setting b), so far there is no notion of mixing that helps describing the systems in our setting.

C) Finally, also the atypical points (those which build a measure zero set) in those systems are supposed to be studied, e.g. by determining the Hausdorff dimension of points showing a particular behaviour.

Some of the results are expected to be obtained mostly by a applying a combination of the methods from settings a) and b). These parts of the project can be considered as a lower risk and in an optimal case give already early publishable results. Moreover, there are a number of number theoretic examples with this behaviour, e.g. different non-standard continued fraction expansions giving nice applications. On the other hand there are also a number of zero-entropy dynamical systems (like suspension flows or adding machines) and together with a non-integrable observable they fall into the above described setting - however for them even in setting a) and b) a lot less is known.

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Project specific entry requirements: Master in Mathematics or a comparable qualification – preference will be given to candidates who can show a background in at least one of the following subjects: measure theory, stochastic processes, ergodic theory, fractal geometry.

Potential PhD programme of study: PhD in Mathematics

Department: Mathematics and Statistics

Location: Harrison or Laver building

Please direct project specific enquiries to: Dr Tanja Schindler

Please ensure you read the entry requirements for the potential programme you are applying for.

To Apply for this project please click on the following link - **APPLY HERE**